Selected Fusion Applications: Defence Domain

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1. **Multiple Hypothesis Tracking: Basic Principles**
2. **Tracking & Management for Phased Array Radar**
3. **GMTI Tracking and Fusion of Context Information**

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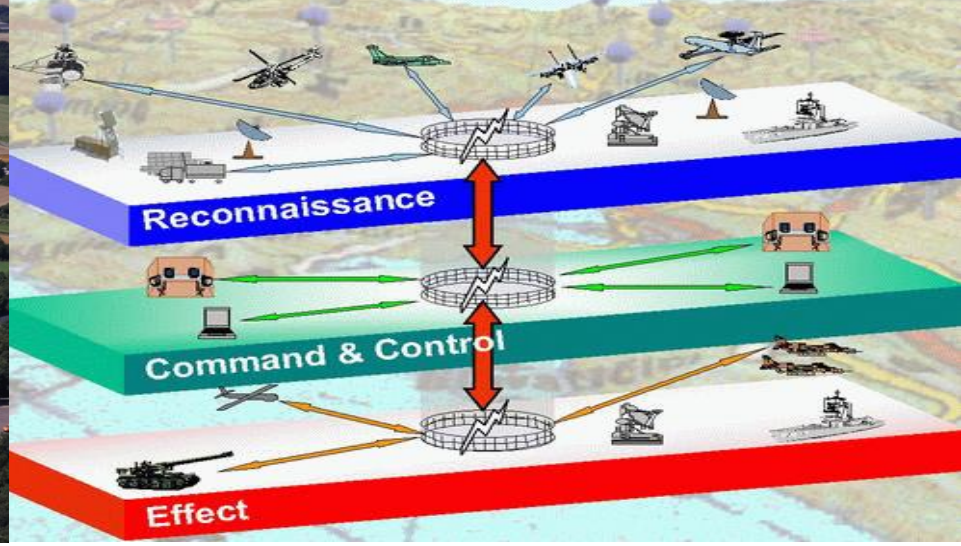

Bonn University
Chair for Communication and Distributed Systems (CS-IV)
Prof. Dr. Peter Martini

FKIE: Fraunhofer Institute for Communications, Information Processing, and Ergonomics
Director: Prof. Dr. Peter Martini, Deputy: Prof. Dr.-Ing. Christopher Schlick

RWTH Aachen
Institute/Chair of Industrial Engin. & Ergonomics (IAW)
Prof. Dr.-Ing. Christopher Schlick

- **Sensor Data and Information Fusion**
- **Communication Systeme**
- **Information Technology for C2-Systems**
- **Ergonomics: HMI, Human Factors**
- **Unmanned Systems**
- **Cyber Defense**

"Institute for Network Enabled Capabilities"

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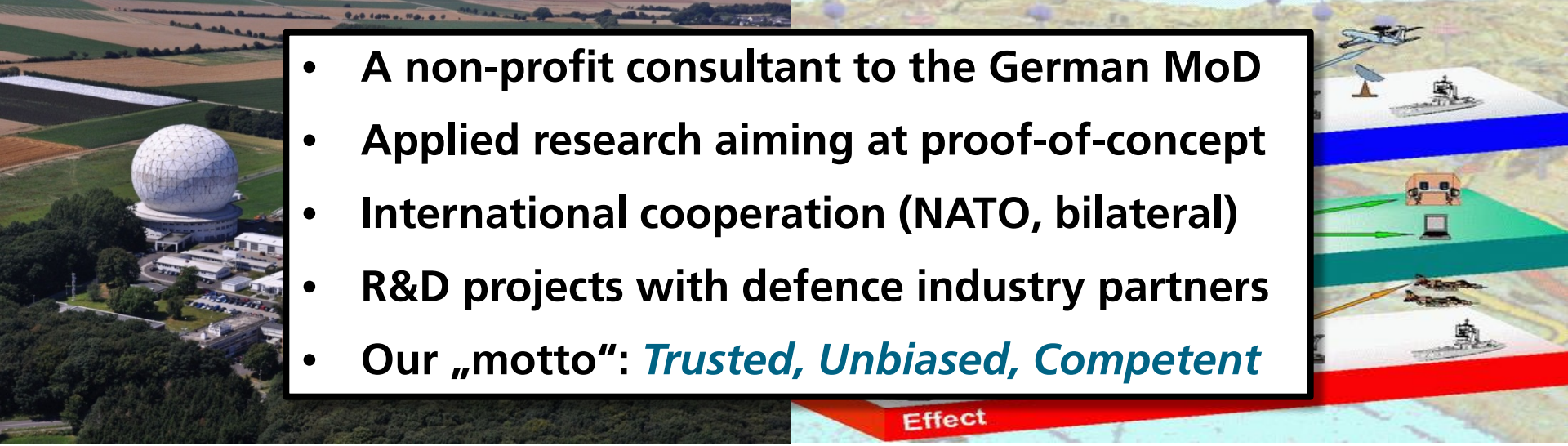

- A non-profit consultant to the German MoD
- Applied research aiming at proof-of-concept
- International cooperation (NATO, bilateral)
- R&D projects with defence industry partners
- Our „motto“: *Trusted, Unbiased, Competent*

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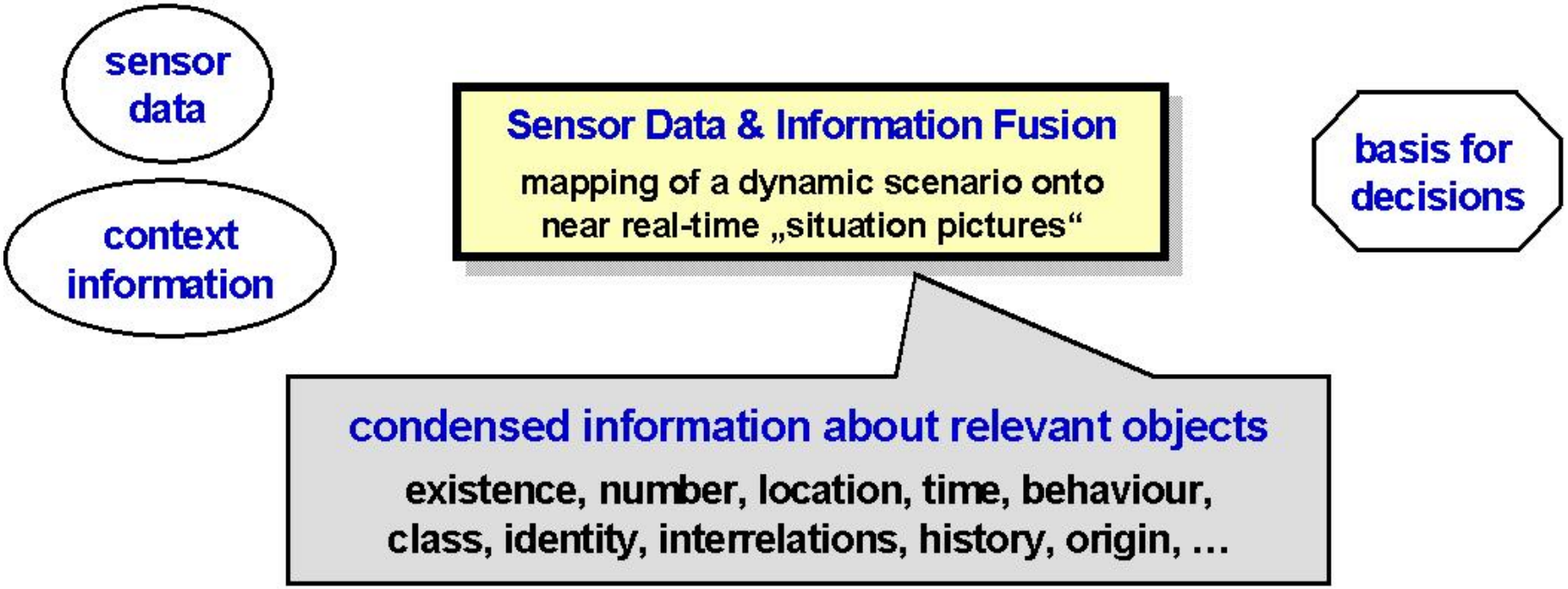
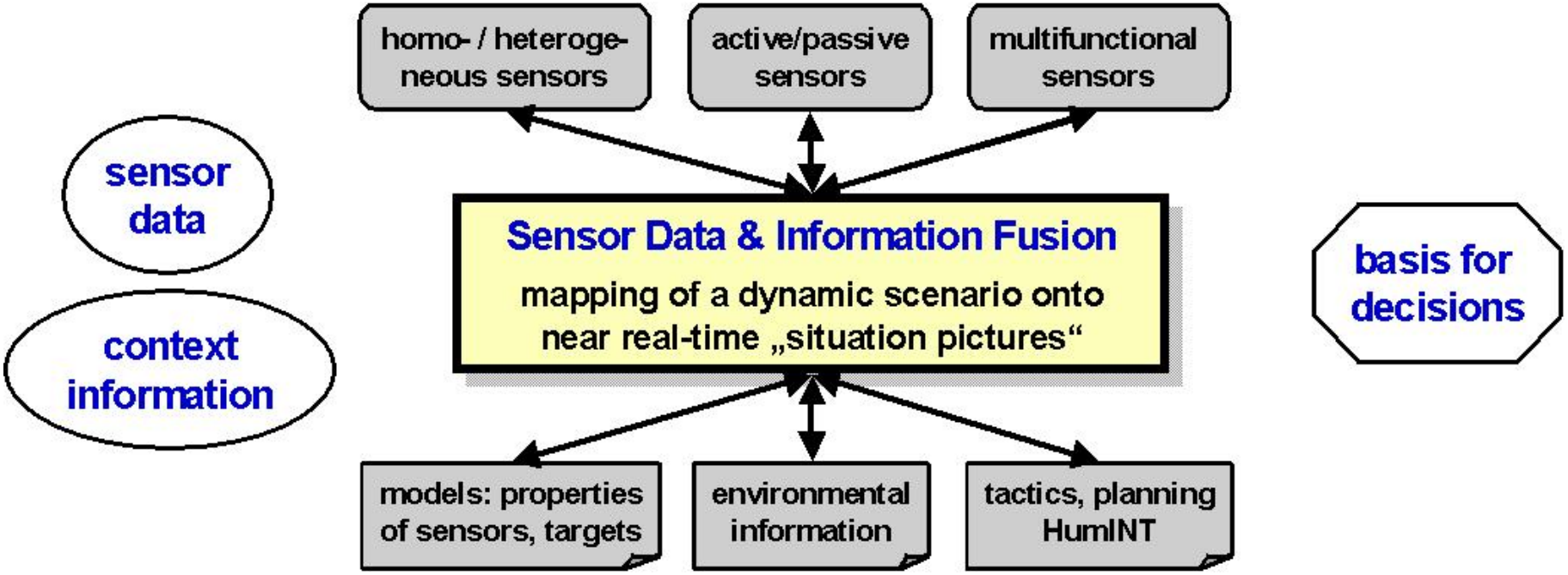
Sensor & Information Fusion: Basic Task



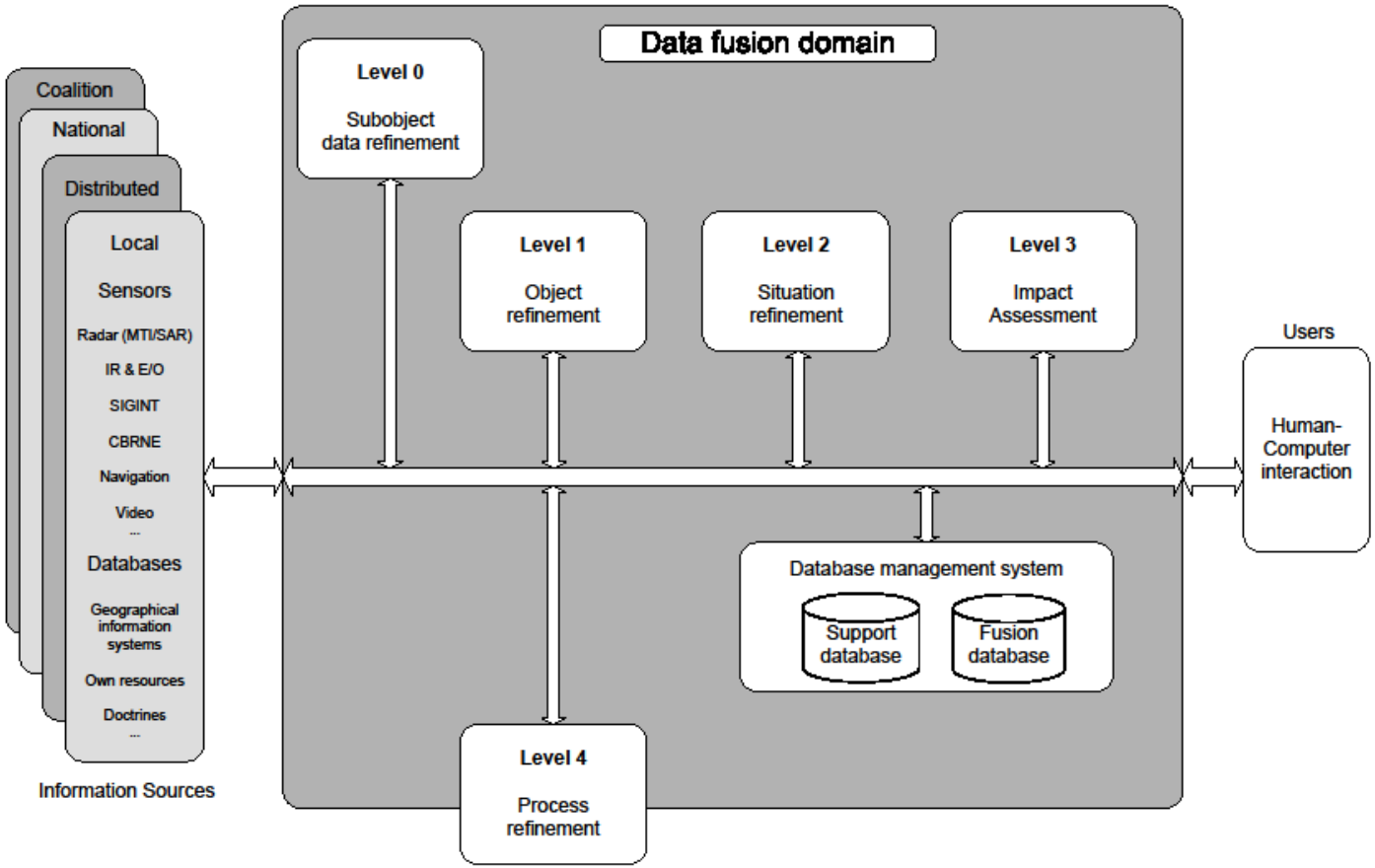
information sources: defined by operational requirements

Sensor & Information Fusion: Basic Task



information to be fused: imprecise, incomplete, ambiguous, un-resolved, false, deceptive, hard to formalize, contradictory . . .

First Scientific Systematization in George Orwell's Year 1984



Joint Directors of Laboratories (JDL): Data Fusion Model

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FKIE-SDF: Sensor Data and Information Fusion

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Data Fusion for Array-Sensors
Ulrich Nickel

- *Multistatics, GSM-based Surveillance*
- *Advanced Array-Signal Processing*

Localization & Navigation
Klaus Wild

- *Emitter Localization & Tracking*
- *Robust Navigation, NAVWAR*

Wide-area Surveillance
Martin Ulmke

- *Airborne Ground Surveillance*
- *Maritime Surveillance*

Threat Recognition
Joachim Biermann

- *Civil Security Applications*
- *Context-based Exploitation*

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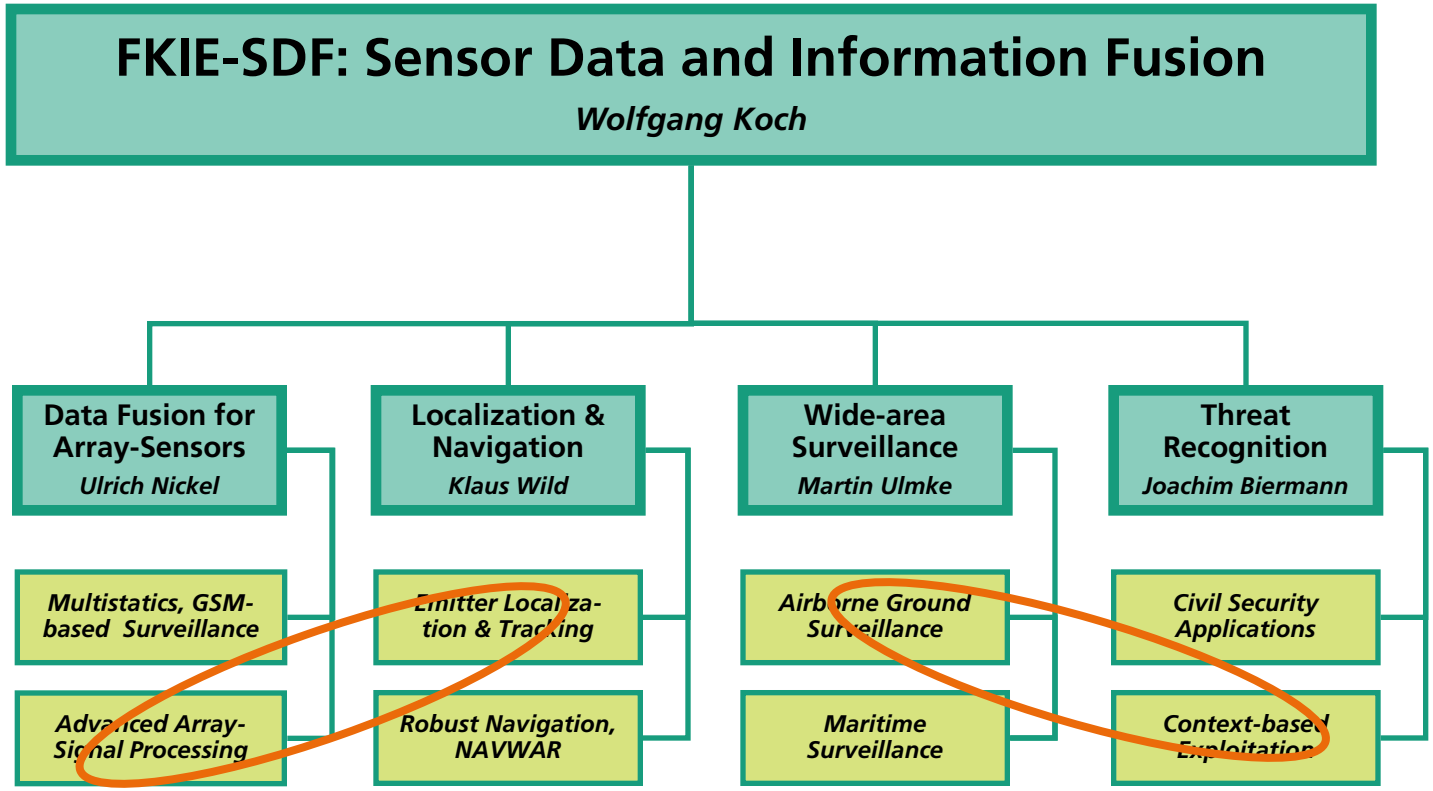

Typically: temporally limited cross-group research projects

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FKIE-SDF: Sensor Data and Information Fusion

Wolfgang Koch

Bonn University

Work Group SDF at CS IV (Prof. Martini)

Priv.-Doz. Dr. Wolfgang Koch

Admin
J. Biermann

Acquisition
K. Wild

Public Calls
G. Luedtke

Lab
M. Okum

Lecture Series
SDF - Methods and Applications

Seminar
Advanced Emerging Topics of SDF

Practicum
Efficient and Robust Algorithms for SDF

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Uni/FKIE-SDF: Master and PhD Student Group – Overarching the Research Groups

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Target Tracking: Basic Idea, Demonstration

Problem-inherent uncertainties and ambiguities!

BAYES: processing scheme for *'soft'*, *'delayed'* decision

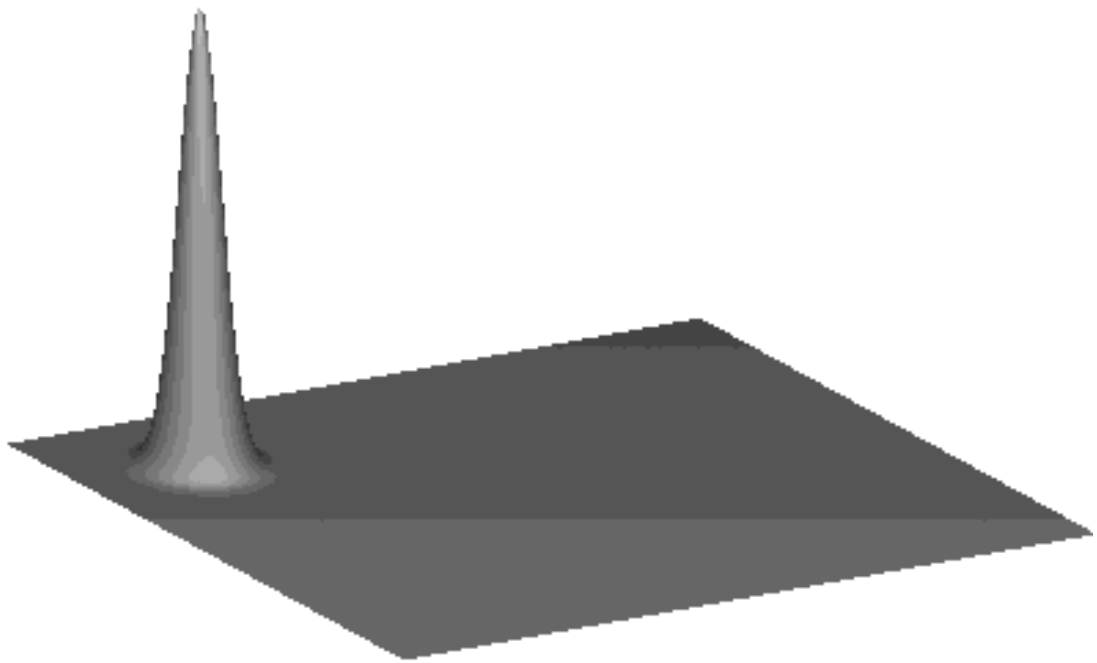
sensor performance: • resolution conflicts • DOPPLER blindness

environment: • dense situations • clutter • jamming/deception

target characteristics: • qualitatively distinct maneuvering phases

background knowledge • vehicles on road *networks* • tactics

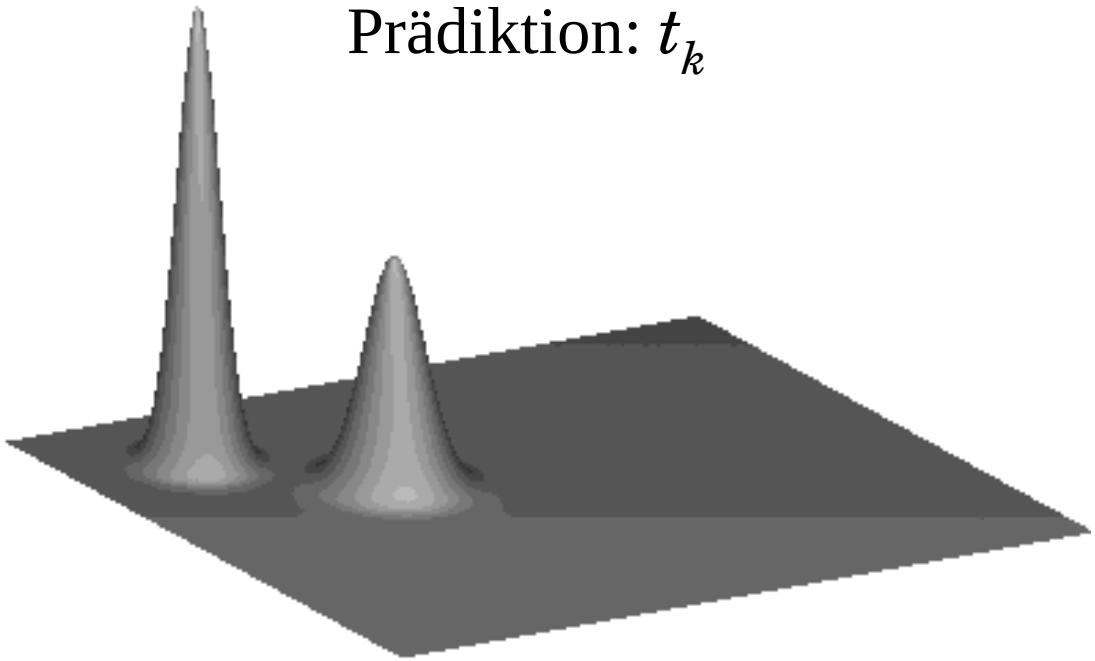
pdf: t_{k-1}



'Probability densities functions (pdf)' $p(\mathrm{x}_{k-1}|\mathcal{Z}^{k-1})$ **represent imprecise knowledge on the 'state'** x_{k-1} **based on imprecise measurements** $\mathcal{Z}^{k-1}$**.**

pdf: t_{k-1}

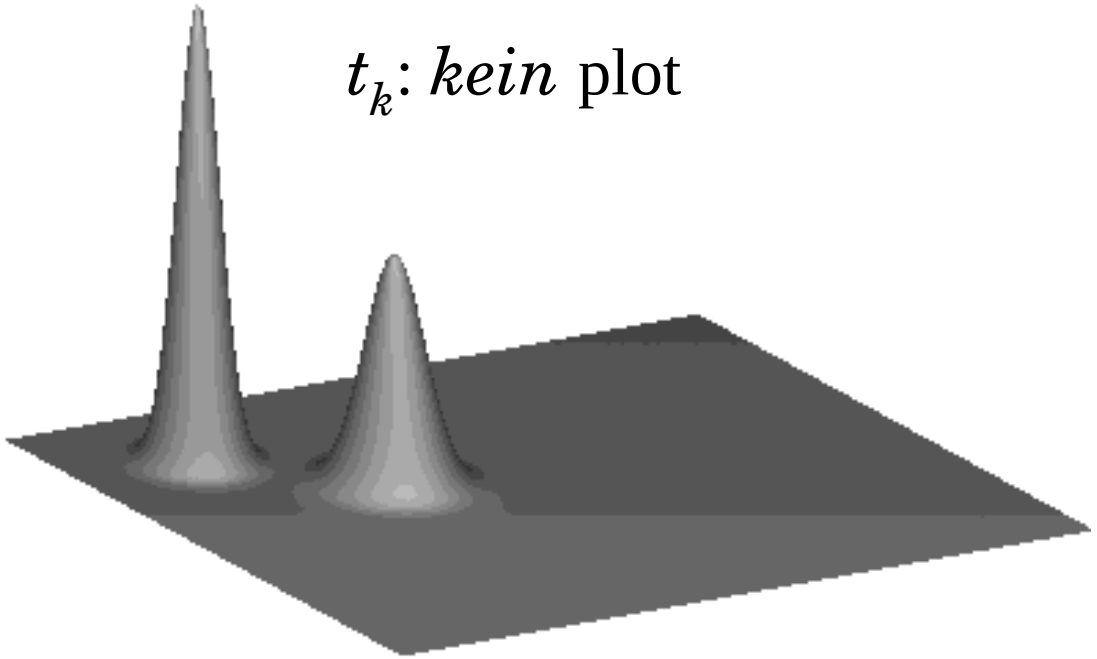
Prädiktion: t_k



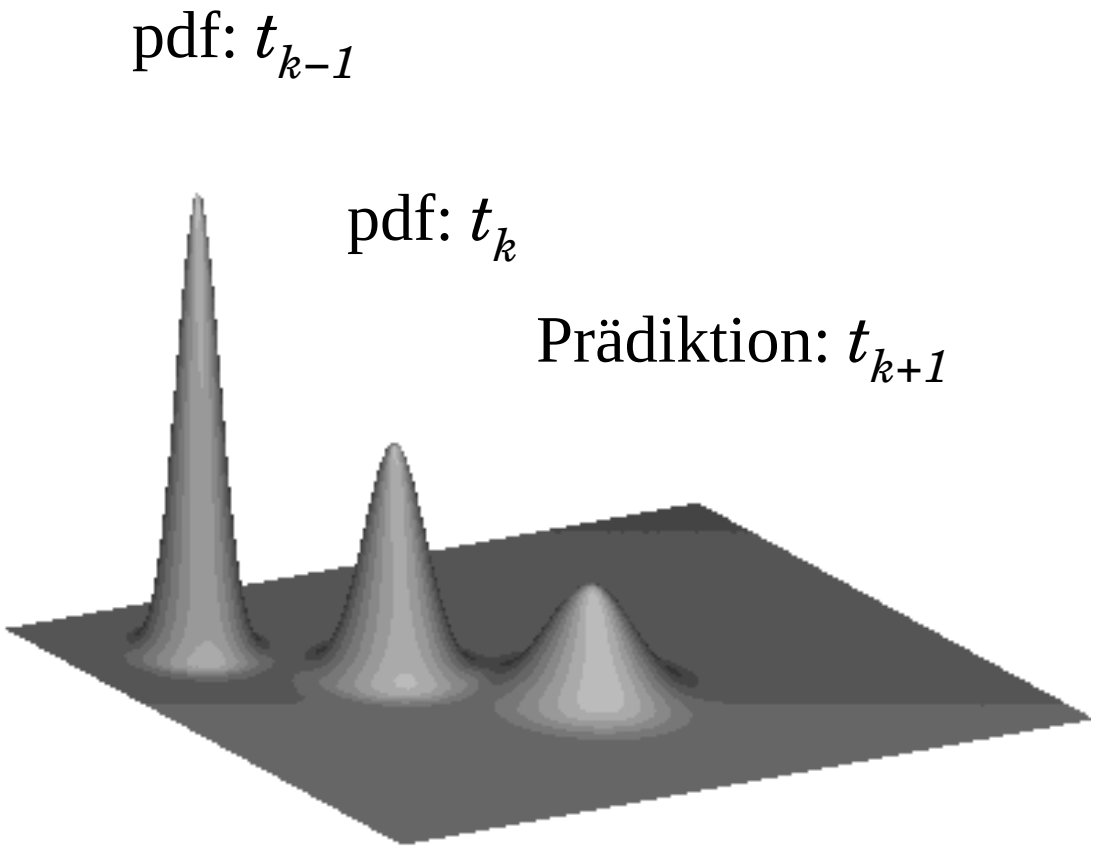
Exploit imprecise knowledge on the dynamical behavior of the object.

$$\underbrace{p(\mathbf{x}_k|\mathcal{Z}^{k-1})}_{\text{prediction}} = \int d\mathbf{x}_{k-1}\ \underbrace{p(\mathbf{x}_k|\mathbf{x}_{k-1})}_{\text{dynamics}}\ \underbrace{p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}_{\text{old knowledge}}.$$

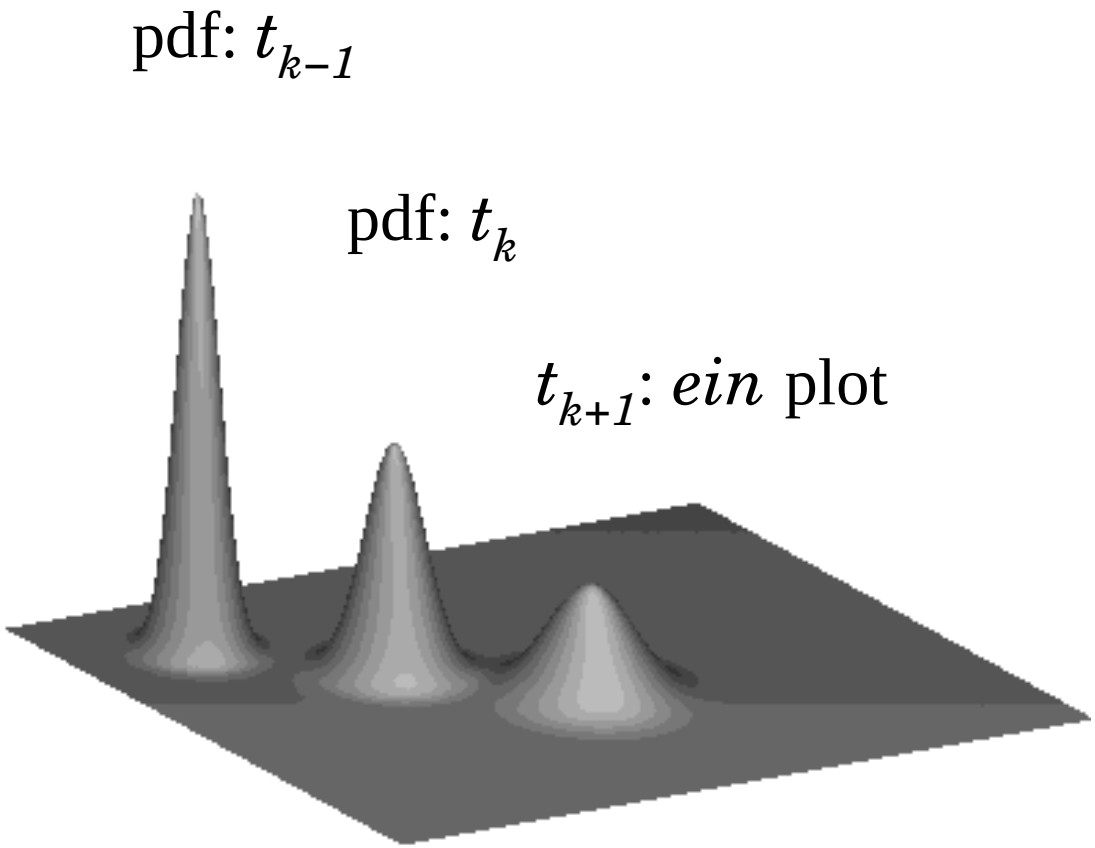
pdf: t_{k-1}



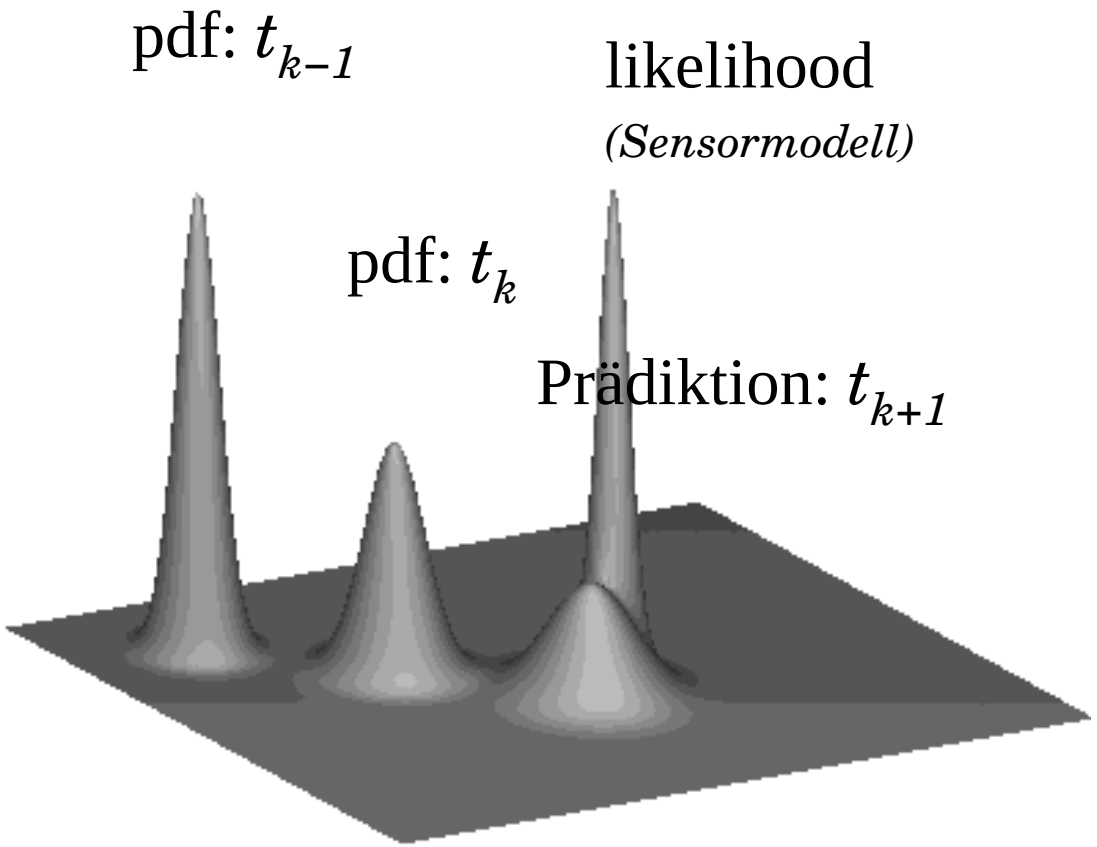
missing sensor detection: ‘data processing’ = prediction
(not always: exploitation of ‘negative’ sensor evidence)



missing sensor information: increasing knowledge dissipation

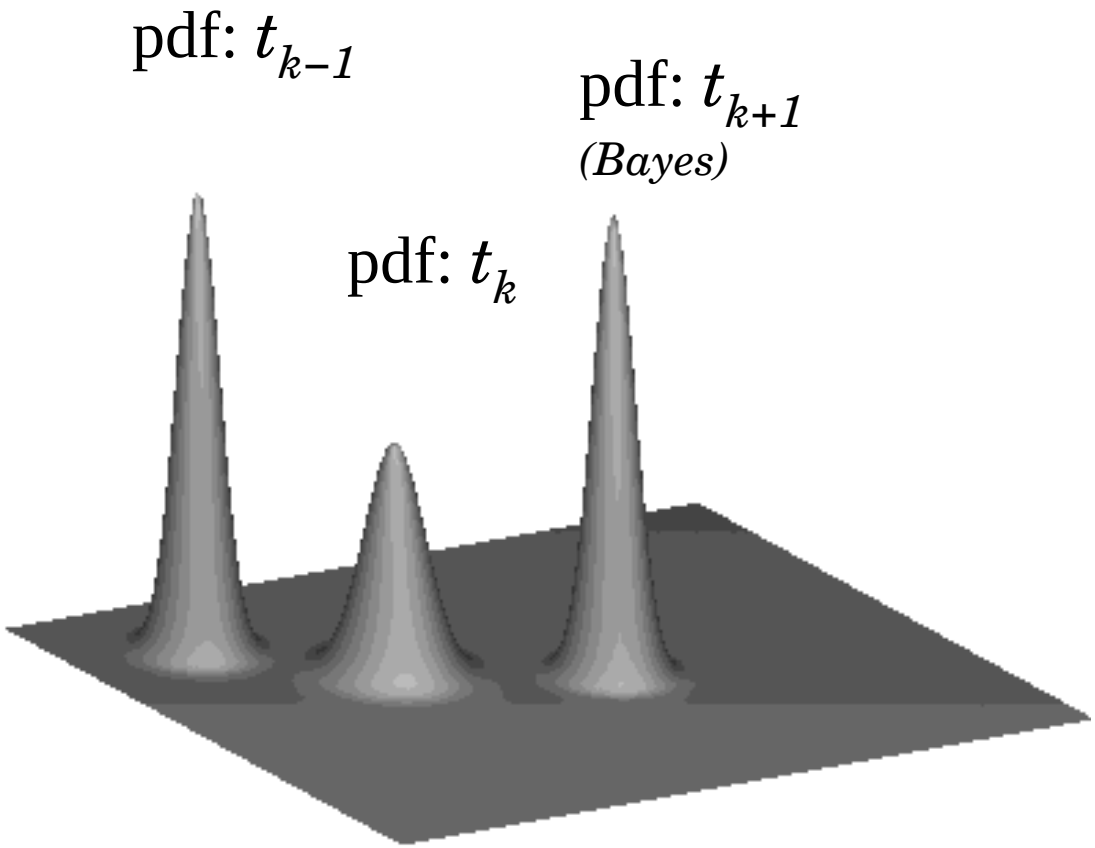


sensor information on the kinematical object state

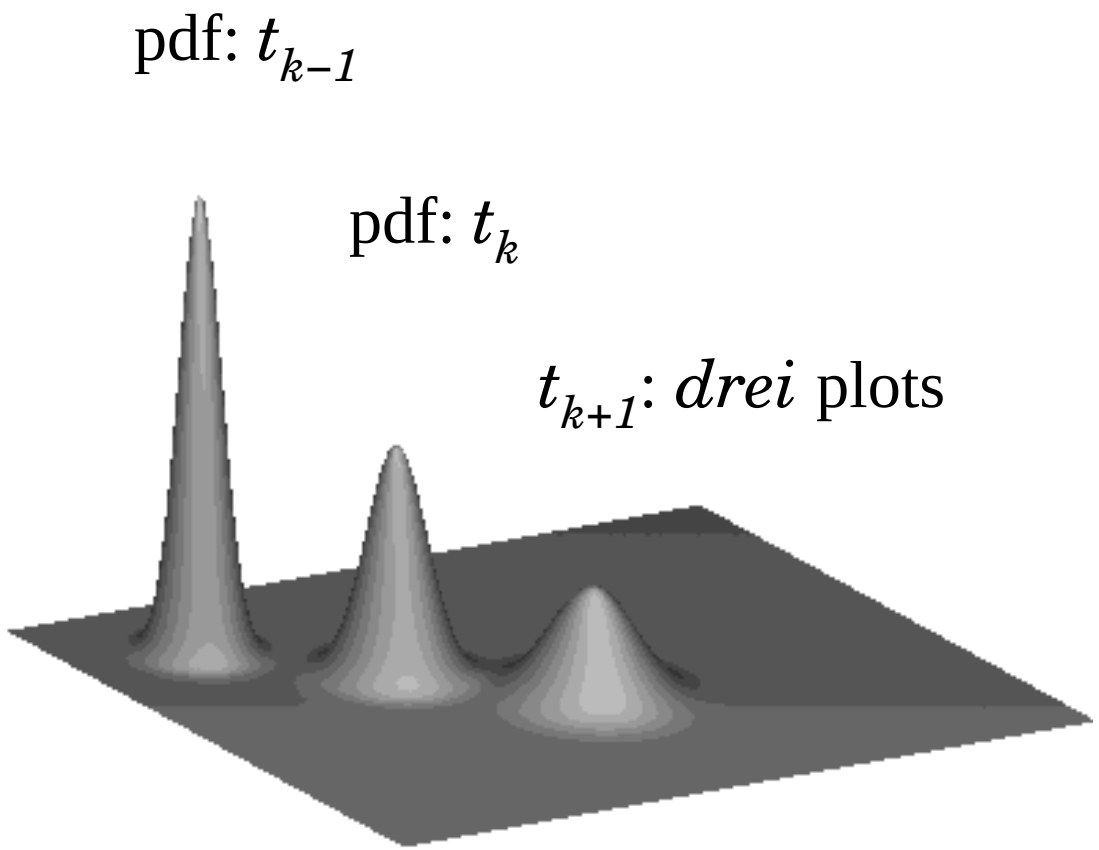


BAYES' formula:

$$\underbrace{p(\mathbf{x}_{k+1}|\mathcal{Z}^{k+1})}_{\text{new knowledge}} = \frac{p(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})\, p(\mathbf{x}_{k+1}|\mathcal{Z}^{k})}{\int d\mathbf{x}_{k+1}\, p(\underbrace{\mathbf{z}_{k+1}}_{\text{plot}}|\mathbf{x}_{k+1})\, \underbrace{p(\mathbf{x}_{k+1}|\mathcal{Z}^{k})}_{\text{prediction}}}$$

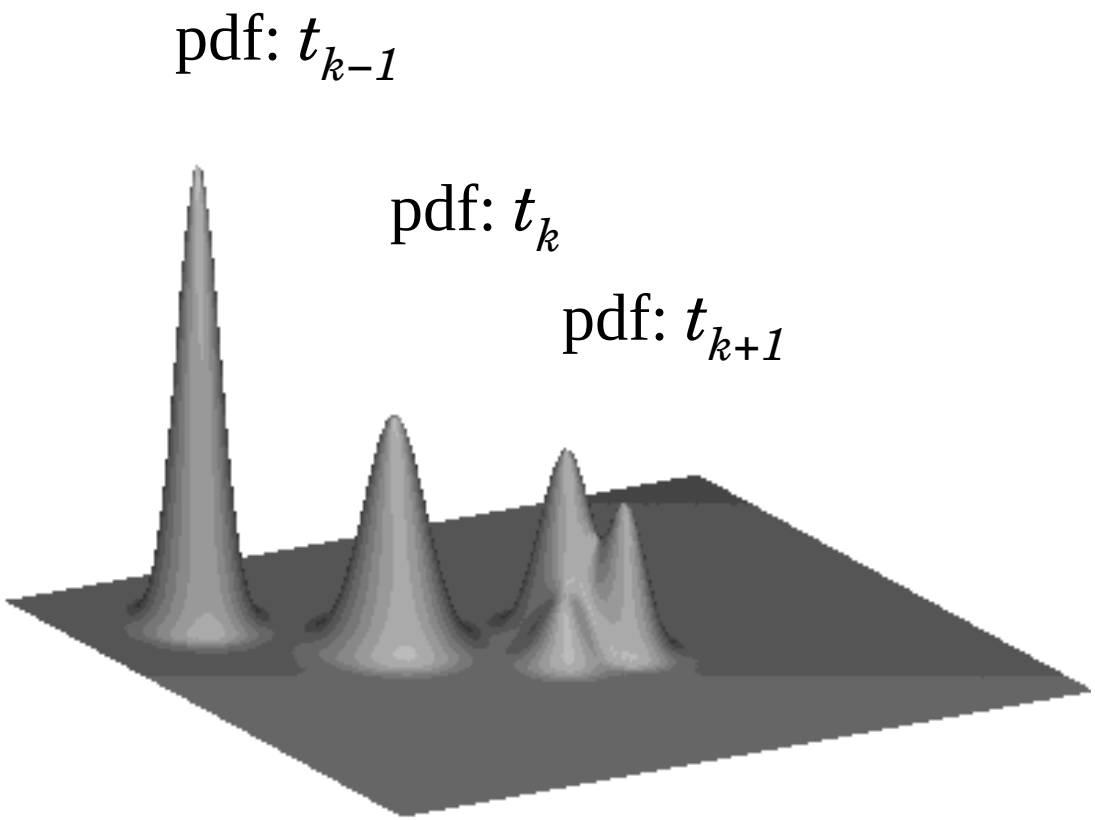


filtering = sensor data processing

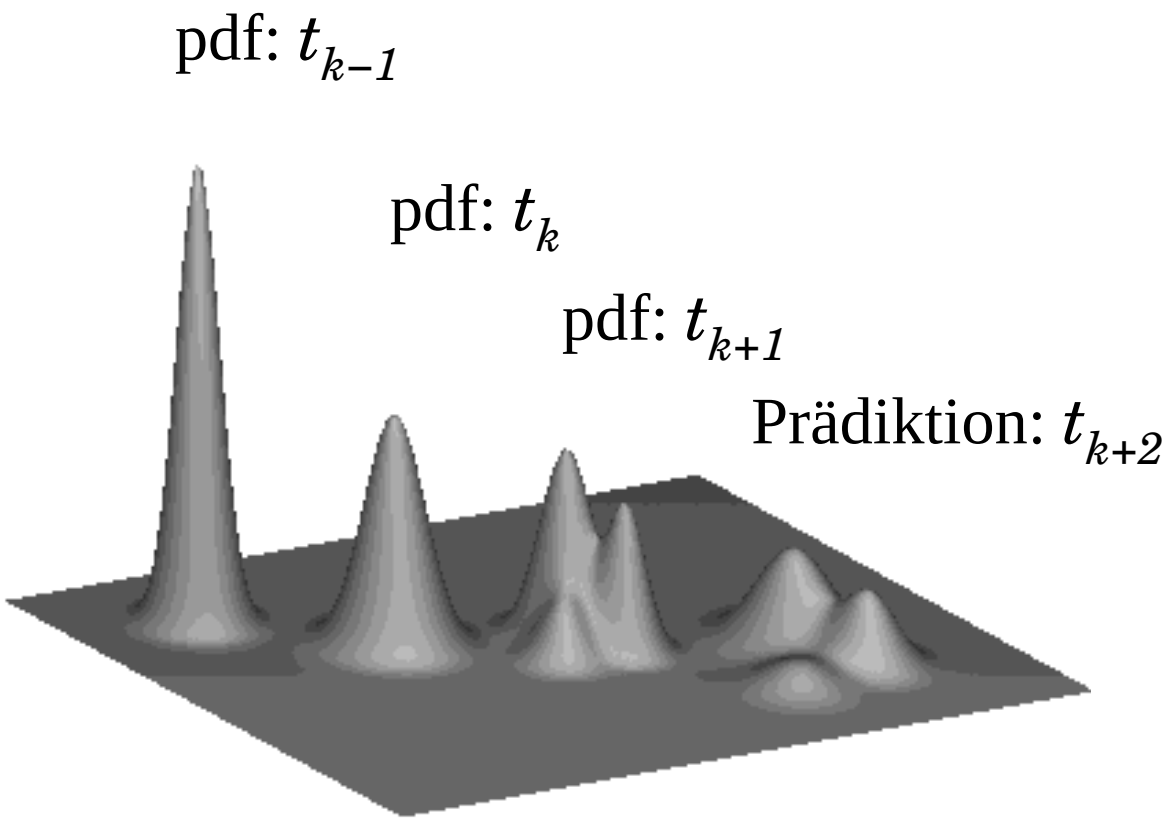


ambiguities by false plots: 1 + 3 data interpretation hypotheses

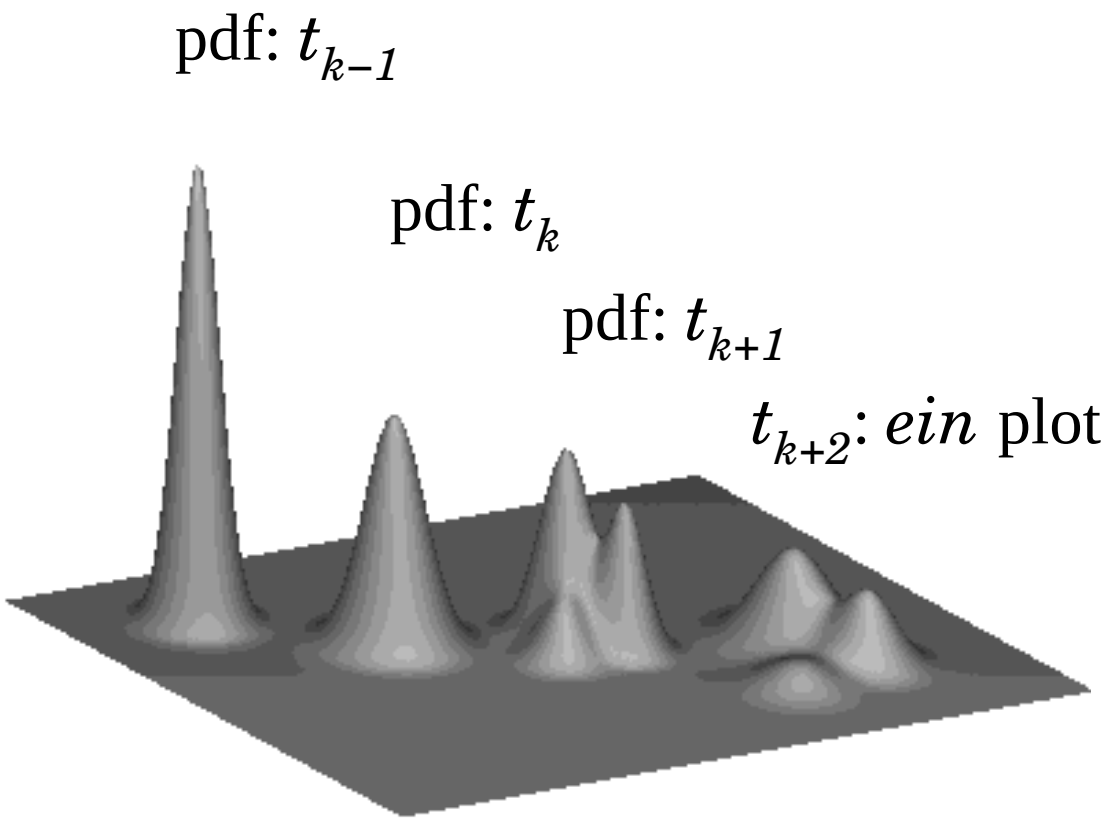
('detection probability', false alarm statistics)



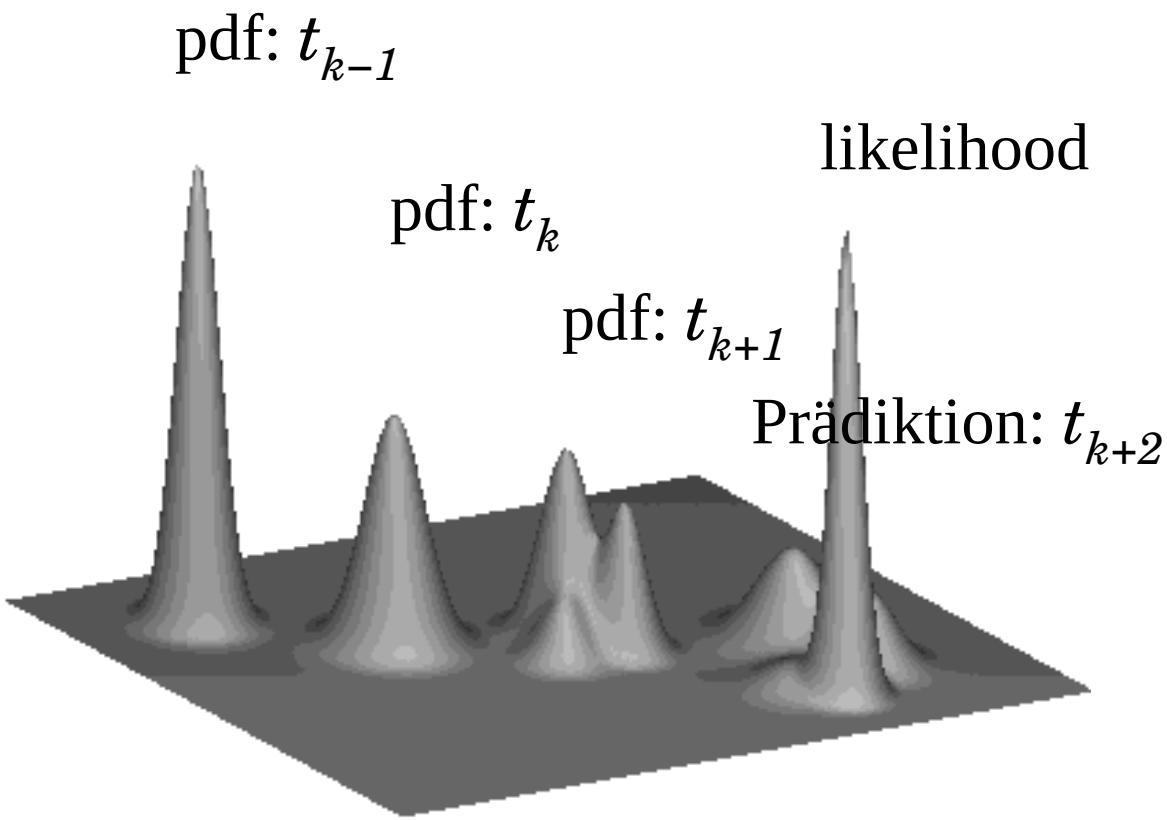
Multimodal pdfs reflect ambiguities inherent in the data.



temporal propagation: dissipation of the probability densities



association tasks: sensor data $\leftrightarrow$ interpretation hypotheses



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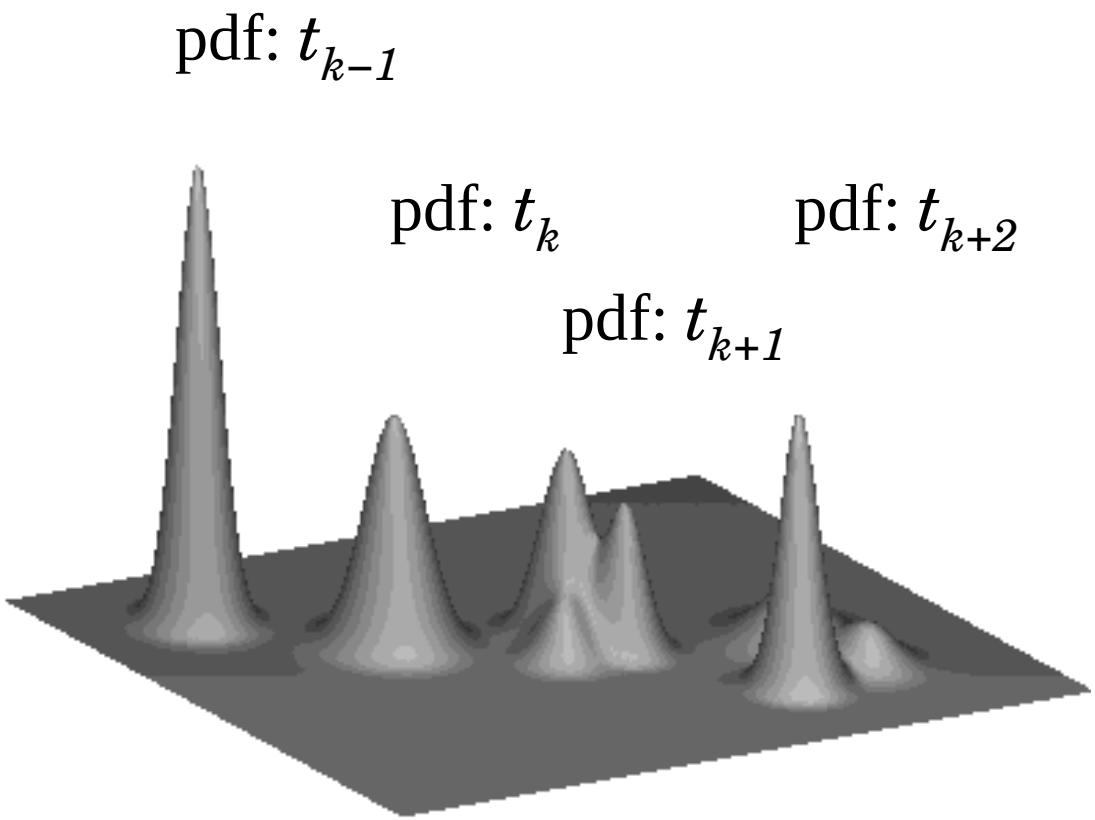
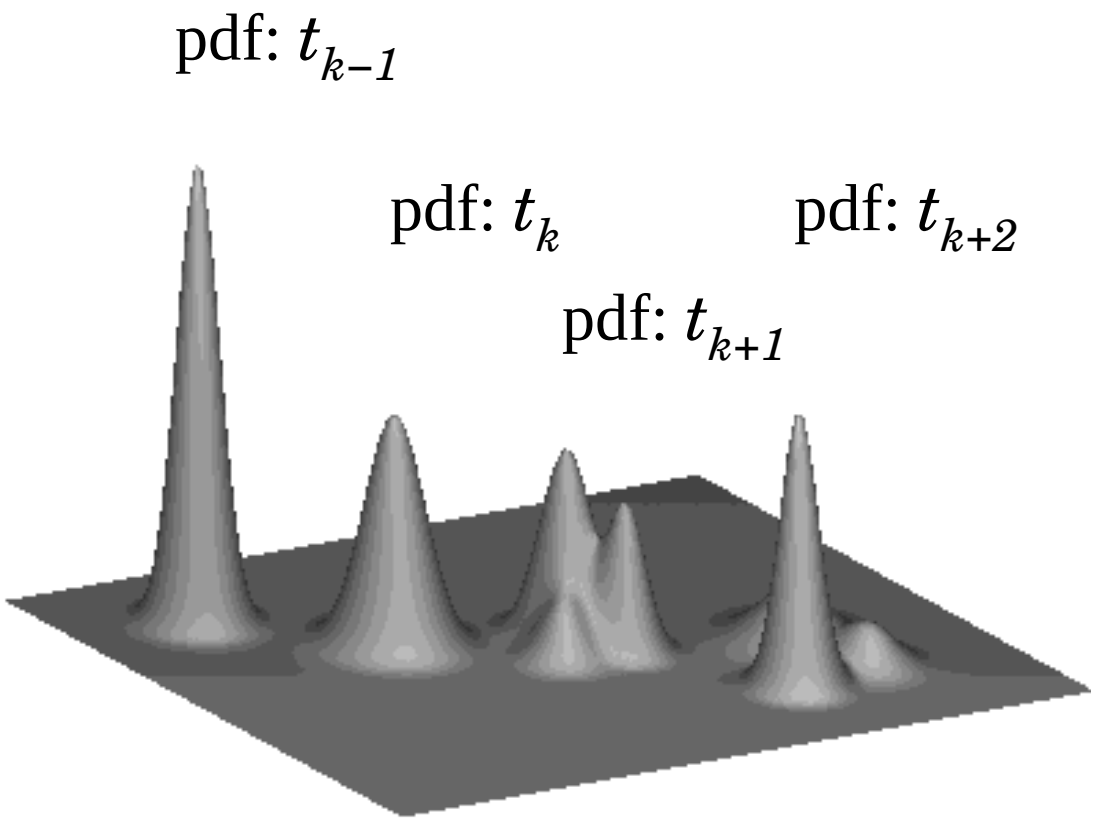
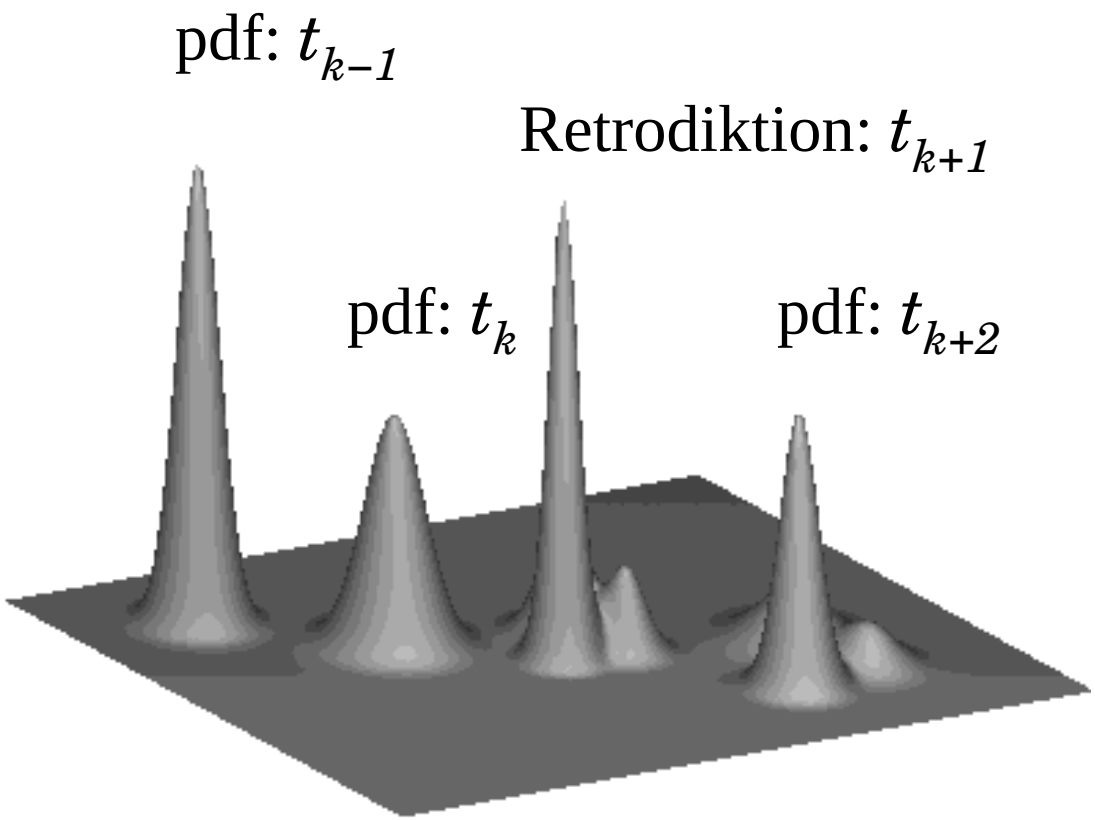


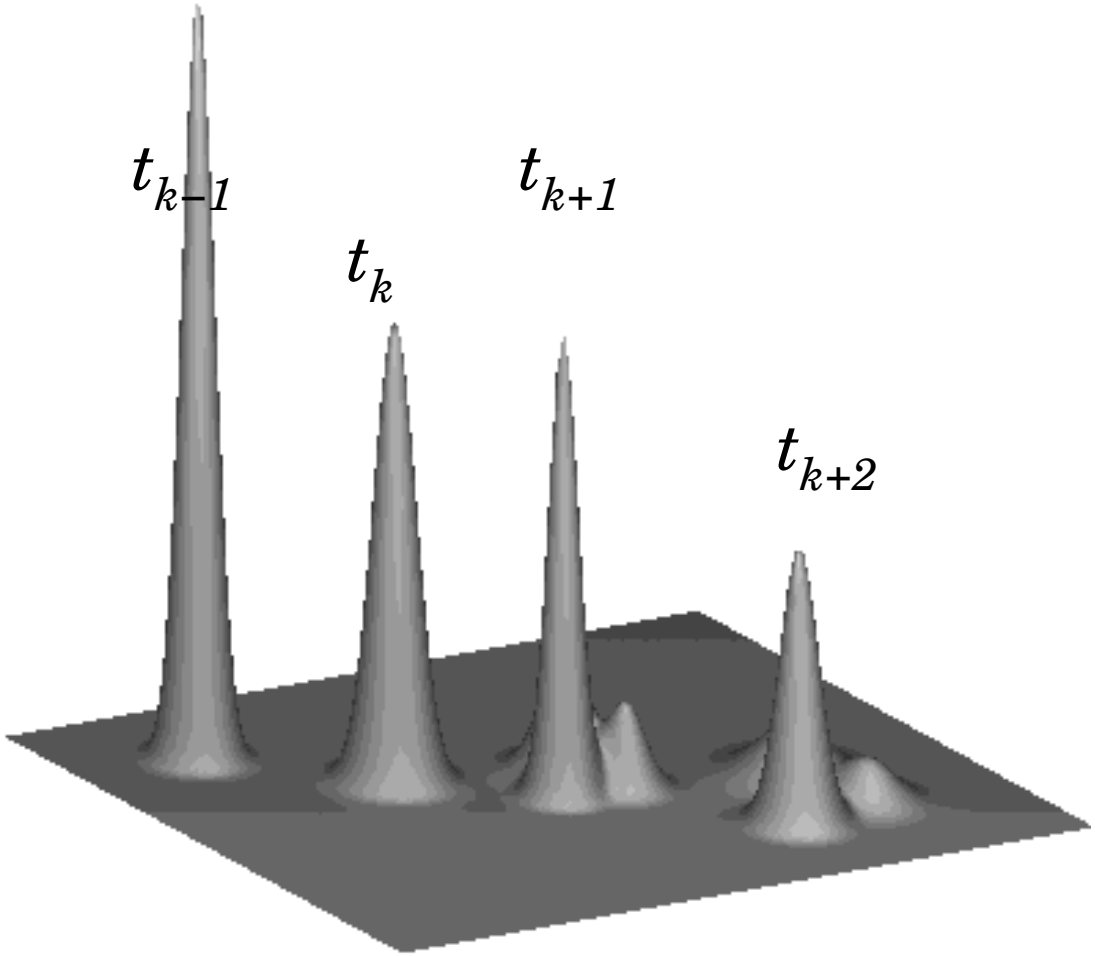
in particular: re-calculation of the hypothesis weights



How does new knowledge affect the knowledge in the past of a past state?



‘retrodiction’: a retrospective analysis of the past



optimal information processing at present and for the past

Multiple Hypothesis Tracking: Basic Idea

Iterative updating of conditional probability densities!

kinematic target state $\mathbf{x}_k$ at time t_k, **accumulated sensor data** $\mathcal{Z}^k$

a priori knowledge: target dynamics models, sensor model, road maps

- **prediction:** $p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \xrightarrow[\text{road maps}]{\text{dynamics model}} p(\mathbf{x}_k|\mathcal{Z}^{k-1})$
- **filtering:** $p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \xrightarrow[\text{sensor model}]{\text{sensor data } Z_k} p(\mathbf{x}_k|\mathcal{Z}^{k})$
- **retrodiction:** $p(\mathbf{x}_{l-1}|\mathcal{Z}^{k}) \xleftarrow[\text{dynamics model}]{\text{filtering output}} p(\mathbf{x}_l|\mathcal{Z}^{k})$

– ***finite mixture:*** **inherent ambiguity (data, model, road *network*)**
– ***optimal estimators:*** **e.g. minimum mean squared error (MMSE)**
– ***track initiation, termination:*** **sequential likelihood ratio testing**

Difficult Military Operational Conditions

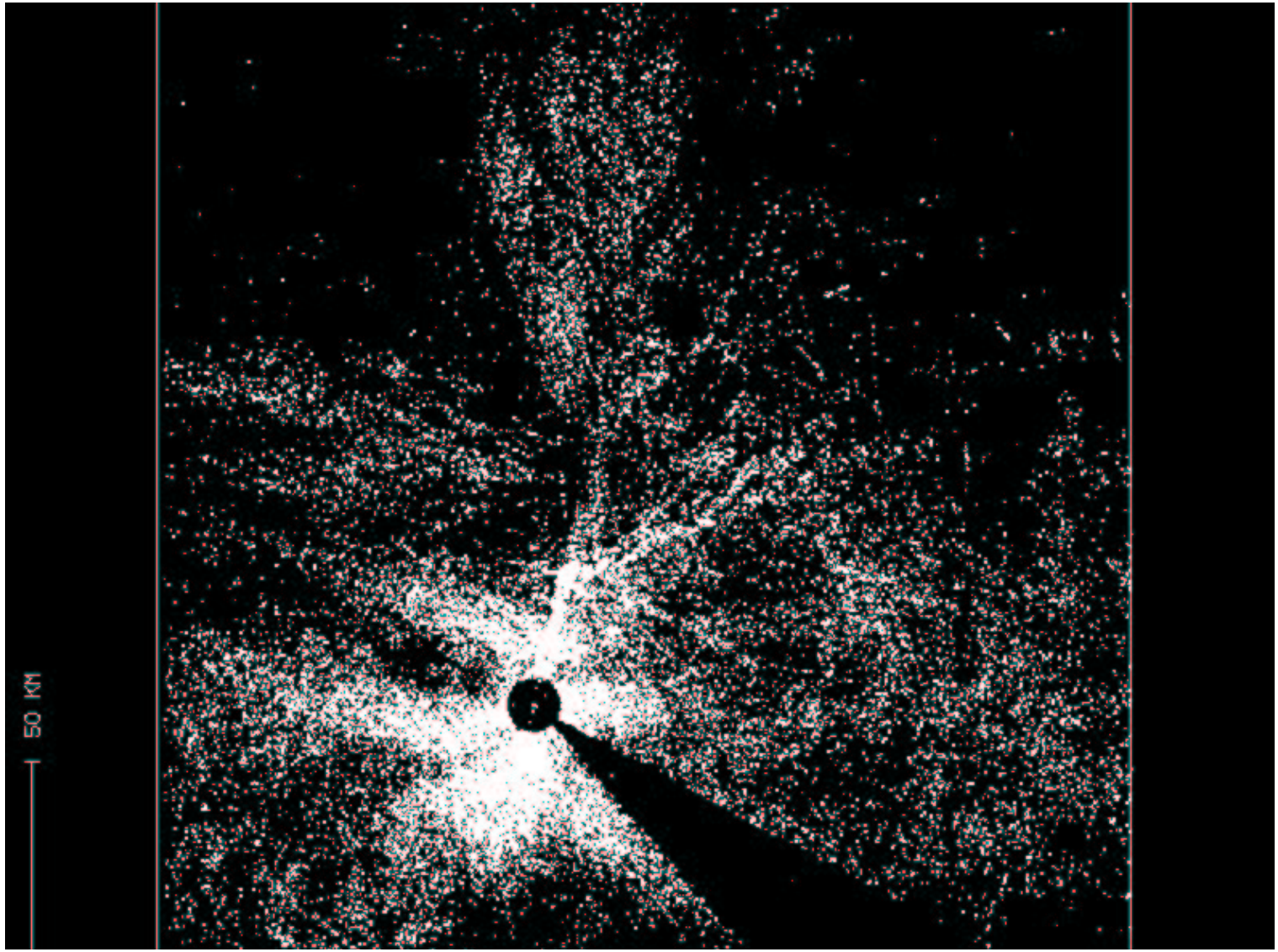
object detection:
- small objects: detection probability $P_D < 1$
- fading: consecutive missing plots (interference)
- moving platforms: minimum detectable velocity

measurements:
- false returns (residual clutter, birds, clouds)
- low data update rates (long-range radar, e.g.)
- measurement errors, overlapping gates

sensor resolution:
- characteristic parameters band-/beam width
- group measurements: resolution probability
- important: *qualitatively* correct modeling

object behavior:
- applications: high maneuvering capability
- qualitatively distinct maneuvering phases
- dynamic object parameters a priori unknown

Demonstration: Multiple Hypothesis Tracking



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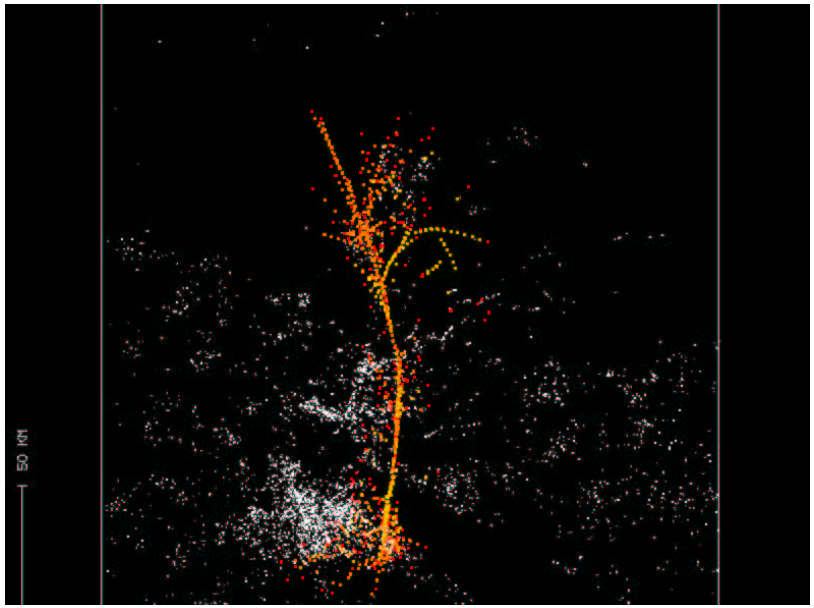
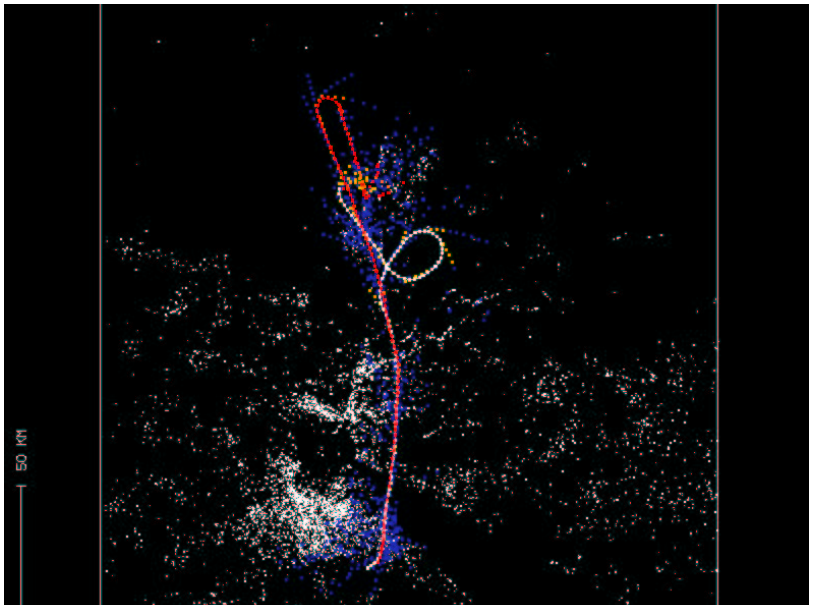
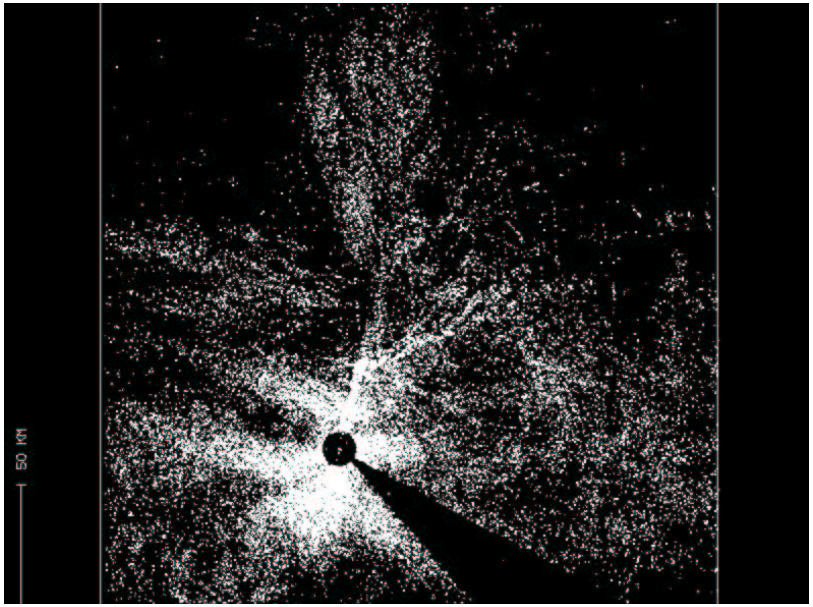
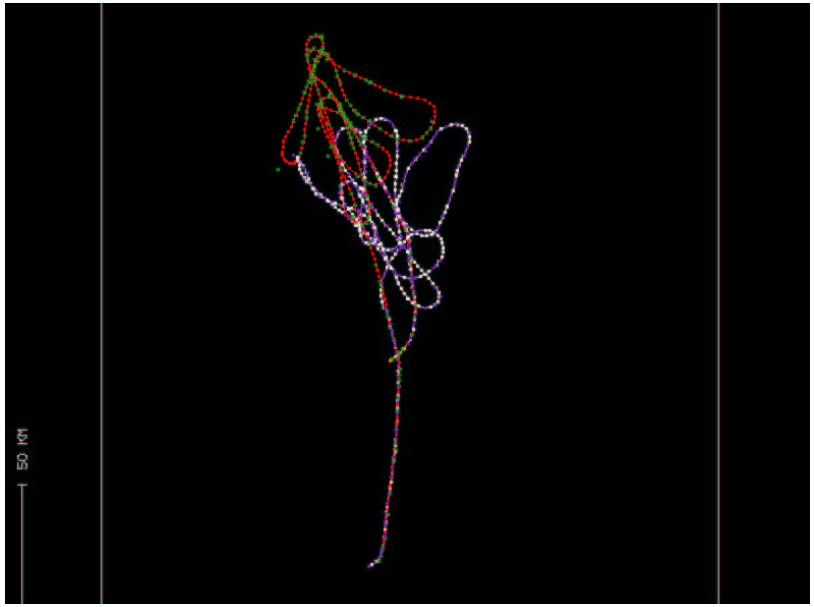


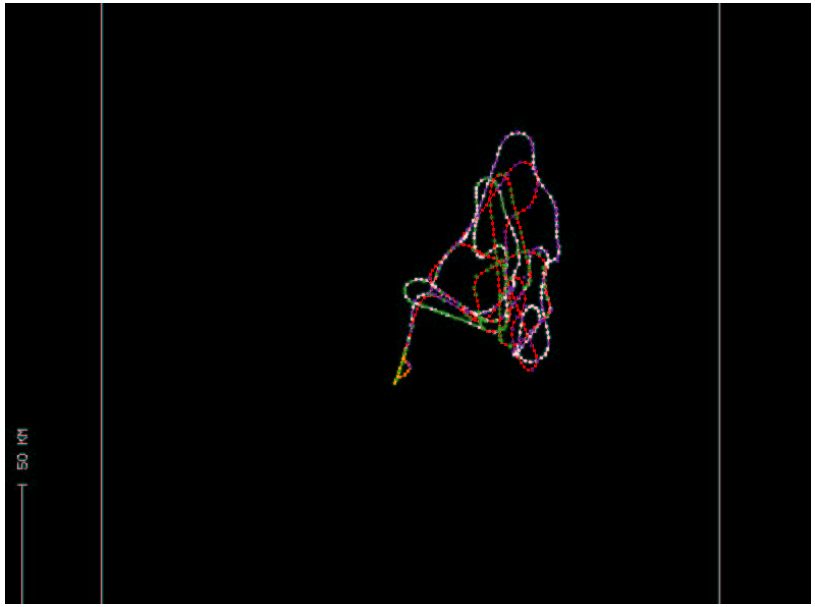


Koch, W., Fixed-Interval Retrodiction Approach to Bayesian IMM-MHT for Maneuvering Multiple Targets. *IEEE Transactions on Aerospace and Electronic Systems, Vol. 36, No. 1, p.2-14, January 2000* .

Koch, W. and van Keuk, G., Multiple Hypothesis Track Maintenance with Possibly Unresolved Measurements. *IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 3, p.883-892, July 1997*.







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Summary: BAYESian (Multi-) Sensor Tracking

- **Basis:** In the course of time one or several sensors produce measurements of targets of interest. Each target is characterized by its current state vector, being expected to change with time.

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- **Approach:** Interpret measurements and target vectors as random variables (RVs). Describe by probability density functions (pdf) what is known about them.
- **Solution:** Derive iteration formulae for calculating the pdfs! Develop a mechanism for initiation! By doing so, exploit all background information available! Derive state estimates from the pdfs along with appropriate quality measures!

How to deal with probability density functions?

- pdf $p(x)$: Extract *probability statements* about the RV x by integration!
- naïvely: *positive* and *normalized* functions ($p(x) \geq 0$, $\int dx\, p(x) = 1$)

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- *conditional pdf* $p(x|y) = \frac{p(x,y)}{p(y)}$: Impact of information on y on RV x?
- *marginal density* $p(x) = \int dy\, p(x,y) = \int dy\, p(x|y)\, p(y)$: Enter y!
- Bayes: $p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{p(y|x)p(x)}{\int dx\, p(y|x)p(x)}$: $p(x|y) \leftarrow p(y|x), p(x)$!

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- *certain knowledge* on x: $p(x) = \delta(x-y)$ '=' $\lim_{\sigma\to 0} \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-\frac{1}{2}\frac{(x-y)^2}{\sigma^2}}$
- *transformed* $RV\, y = t[x]$: $p(y) = \int dx p(y,x) = \int dx p(y|x) p_x(x) =$ $\int dx\, \delta(y - t[x])\, p_x(x) =: [\mathcal{T} p_x](y)$ ($\mathcal{T} : p_x \mapsto p$, "transfer operator")

The Multivariate GAUSSian Pdf

- *wanted:* probabilities 'concentrated' around a center $\mathbf{x}$

- *quadratic distance:* $q(x) = \frac{1}{2}(x - \mathbf{x})\mathbf{P}^{-1}(x - \mathbf{x})^\top$

  $q(x)$ defines an ellipsoid around $\mathbf{x}$, its volume and orientation being determined by a matrix $\mathbf{P}$ (symmetric: $\mathbf{P}^\top = \mathbf{P}$, positively definite: all eigenvalues > 0).

- *first attempt:* $p(x) = \mathrm{e}^{-q(x)} / \int dx\, \mathrm{e}^{-q(x)}$ (normalized!)

$$\boxed{p(x) = \mathcal{N}(x;\, \mathbf{x},\, \mathbf{P}) = \frac{1}{\sqrt{|2\pi\mathbf{P}|}}\, \mathrm{e}^{-\frac{1}{2}(x-\mathbf{x})^\top \mathbf{P}^{-1}(x-\mathbf{x})}}$$

$\mathbb{E}[x] = \mathbf{x},\ \mathbb{E}[(x - \mathbf{x})(x - \mathbf{x})^\top] = \mathbf{P}$ (covariance matrix)

- GAUSS*ian Mixtures:* $p(x) = \sum_i p_i\, \mathcal{N}(x;\, \mathbf{x}_i,\, \mathbf{P}_i)$ (weighted sums)

More Precise Formulation of the BAYESian Approach

Consider a set of measurements $Z_l = \{\mathbf{z}_l^j\}_{j=1}^{m_l}$ of a single or a multiple target state $\mathbf{x}_l$ at time instants t_l, $l = 1, \ldots, k$ and the time series:

$$\mathcal{Z}^k = \{Z_k, m_k, Z_{k-1}, m_{k-1}, \ldots, Z_1, m_1\} = \{Z_k, m_k, \mathcal{Z}^{k-1}\}!$$

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Based on $\mathcal{Z}^k$, what can be learned about the object states $\mathbf{x}_l$ at $t_1, \ldots, t_k, t_{k+1}, \ldots$, i.e. for the past, present, and future?

Evidently the answer is given be calculating the pdf $\boxed{p(\mathbf{x}_l|\mathcal{Z}^k)}$!

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multiple sensor measurement fusion: Calculate $p(\mathbf{x}|\mathcal{Z}_1^k, \ldots, \mathcal{Z}_N^k)$!

- communication lines
- common coordinate system: sensor registration

How to calculate the pdf $p(\mathbf{x}_l|\mathcal{Z}^k)$?

Consider at first the present time: $l = k$.

an observation:

BAYES' rule: $$p(\mathbf{x}_k|\mathcal{Z}^k) = p(\mathbf{x}_k|Z_k, m_k, \mathcal{Z}^{k-1})$$

$$= \frac{p(Z_k, m_k|\mathbf{x}_k, \mathcal{Z}^{k-1})\, p(\mathbf{x}_k|\mathcal{Z}^{k-1})}{\int d\mathbf{x}_k\, p(Z_k, m_k|\mathbf{x}_k, \mathcal{Z}^{k-1})\, p(\mathbf{x}_k|\mathcal{Z}^{k-1})}$$

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- $p(\mathbf{x}_k|\mathcal{Z}^{k-1})$ is a *prediction* of the target state at time t_k based on all measurements in the *past*.

- $p(Z_k, m_k|\mathbf{x}_k) \propto \ell(\mathbf{x}_k; Z_k, m_k)$ describes, what the *current* sensor output Z_k, m_k can say about the current target state $\mathbf{x}_k$ and is called *likelihood function*.

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$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k, \mathbf{x}_{k-1}|\mathcal{Z}^{k-1}) \quad \text{marginal pdf}$$

$$= \int d\mathbf{x}_{k-1}\, \underbrace{p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathcal{Z}^{k-1})}_{\text{object dynamics!}}\, \underbrace{p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}_{\text{idea: iteration!}} \quad \text{notion of a conditional pdf}$$

often: $p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathcal{Z}^{k-1}) = p(\mathbf{x}_k|\mathbf{x}_{k-1})$ (MARKOV)

sometimes: $p(\mathbf{x}_k, i_k|\mathbf{x}_{k-1}, i_{k-1}) = \underbrace{p_{i_k,i_{k-1}}}_{\text{phase transition}}\, \mathcal{N}\big(\mathbf{x}_k;\, \underbrace{\mathbf{F}^{i_k}_{k|k-1}}_{\text{deterministic}}\mathbf{x}_{k-1},\, \underbrace{\mathbf{D}^{i_k}_{k|k-1}}_{\text{random}}\big)$ (IMM)

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iteration formula: $$p(\mathbf{x}_k|\mathcal{Z}^k) = \frac{\ell(\mathbf{x}_k; Z_k, m_k) \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k|\mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}{\int d\mathbf{x}_k\, \ell(\mathbf{x}_k; Z_k, m_k) \int d\mathbf{x}_{k-1}\, p(\mathbf{x}_k|\mathbf{x}_{k-1})\, p(\mathbf{x}_{k-1}|\mathcal{Z}^{k-1})}$$

How to calculate the pdf $p(\mathbf{x}_l|\mathcal{Z}^k)$?

Consider the past: $l < k$!

an observation:

$$p(\mathbf{x}_l|\,\mathcal{Z}^k) = \int d\mathbf{x}_{l+1}\, p(\mathbf{x}_l, \mathbf{x}_{l+1}|\,\mathcal{Z}^k)$$

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- $p(\mathbf{x}_{l+1}|\mathcal{Z}^k)$ retrodiction: known from last iteration step
- $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ dynamic object behavior (possibly multiple models)
- $p(\mathbf{x}_l|\mathcal{Z}^l)$ filtering at the time where retrodiction is considered
- GAUSSians or GAUSSian mixtures: Exploit product formula!
- linear GAUSSian likelihood/dynamics: Rauch-Tung-Striebel smoothing

In certain applications *accumulated* state vectors are to be considered.

Object state vectors accumulated over a time window $t_k, t_{k-1}, \ldots, t_n$:

$$\mathbf{x}_{k:n} = (\mathbf{x}_k, \ldots, \mathbf{x}_n)$$

The full information on $\mathbf{x}_{k:n}$ based on a time series of sensor data $\mathcal{Z}^k$ is contained in the corresponding *accumulated state density (ASD):*

$$p(\mathbf{x}_k, \ldots, \mathbf{x}_n | \mathcal{Z}^k) = p(\mathbf{x}_{k:n} | \mathcal{Z}^k).$$

ASDs can be a useful notion in several applications. Examples are:

- a more comprehensive treatment of issues in particle filtering,
- Q-functions in PMHT applications, which are essentially ASDs,
- dealing with out-of-sequence measurements in sensor networks.

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Via marginalizing over all object states in the window, excepting $\mathbf{x}_l$, $n \leq l \leq k$,

$$\int \mathrm{d}\mathbf{x}_k, \ldots, \mathrm{d}\mathbf{x}_{l+1}, \mathrm{d}\mathbf{x}_{l-1}, \ldots, \mathrm{d}\mathbf{x}_n \, p(\mathbf{x}_k, \ldots, \mathbf{x}_n | \mathcal{Z}^k) = p(\mathbf{x}_l | \mathcal{Z}^k),$$

we obtain the pdfs for filtering & retrodiction; ASDs thus unify these notions.

In addition, all correlations between different instants of time are contained.

Bayes' Theorem: a recursion formula for updating ASDs:

$$\boxed{p(\mathbf{x}_{k:n} | \mathcal{Z}^k) = \frac{\ell(\mathbf{x}_k; Z_k) \, p(\mathbf{x}_k | \mathbf{x}_{k-1}) \, p(\mathbf{x}_{k-1:n} | \mathcal{Z}^{k-1})}{\int \mathrm{d}\mathbf{x}_{k:n} \, \ell(\mathbf{x}_k; Z_k) \, p(\mathbf{x}_k | \mathbf{x}_{k-1}) \, p(\mathbf{x}_{k-1:n} | \mathcal{Z}^{k-1})}}$$

A little formalistically speaking, 'sensor data processing' means:

Make sure that the sensor data are no longer explicitly present.

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Closed-form Representations for ASDs

Example: conditions, where Kalman filtering is applicable

likelihood function: $\ell(\mathrm{x}_k; Z_k) = \mathcal{N}(\mathrm{z}_k;\, \mathrm{H}_k \mathrm{x}_k,\, \mathrm{R}_k)$

evolution model: $p(\mathrm{x}_k | \mathrm{x}_{k-1}) = \mathcal{N}(\mathrm{x}_k;\, \mathrm{F}_{k|k-1} \mathrm{x}_{k-1},\, \mathrm{D}_{k|k-1})$

Induction argument (tedious, but elementary calculations):

The corresponding ASD is a Gaussian:

$$p(\mathrm{x}_{k:n} | Z^k) = \mathcal{N}(\mathrm{x}_{k:n};\, \mathrm{x}^k_{k:n},\, \mathrm{P}^k_{k:n})$$

with $\mathrm{x}^k_{k:n} = (\mathrm{x}^\top_{k|k}, \mathrm{x}^\top_{k-1|k}, \ldots, \mathrm{x}^\top_{n|k})^\top$.

$\mathrm{x}_{k|k}$**: standard Kalman filtering,** $\mathrm{x}_{l|k}$**,** $l < k$**: standard RTS smoothing**

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Block Matrix Representation of the ASD Covariance

$$\mathbf{P}^k_{k:n} =$$

$$\begin{pmatrix} \mathbf{P}_{k|k} & \mathbf{P}_{k|k}\mathbf{W}^\top_{k-1|k} & \mathbf{P}_{k|k}\mathbf{W}^\top_{k-2|k} & \cdots & \mathbf{P}_{k|k}\mathbf{W}^\top_{1|k} \\ \mathbf{W}_{k-1|k}\mathbf{P}_{k|k} & \mathbf{P}_{k-1|k} & \mathbf{P}_{k-1|k}\mathbf{W}^\top_{k-2|k-1} & * & \mathbf{P}_{k-1|k}\mathbf{W}^\top_{1|k-1} \\ \mathbf{W}_{k-2|k}\mathbf{P}_{k|k} & \mathbf{W}_{k-2|k-1}\mathbf{P}_{k-1|k} & \mathbf{P}_{k-2|k} & * & \vdots \\ \vdots & * & * & * & \mathbf{P}_{2|k}\mathbf{W}^\top_{1|2} \\ \mathbf{W}_{1|k}\mathbf{P}_{k|k} & \mathbf{W}_{1|k-1}\mathbf{P}_{k-1|k} & \cdots & \mathbf{W}_{1|2}\mathbf{P}_{2|k} & \mathbf{P}_{1|k} \end{pmatrix}$$

$\mathrm{P}_{k|k}$: **standard Kalman filtering,** $\mathrm{P}_{l|k}$, $l < k$: **standard RTS smoothing**

with: $\mathrm{W}_{l|k} = \prod_{\lambda=l}^{k-1} \mathrm{P}_{\lambda|\lambda}\mathrm{F}^\top_{\lambda+1|\lambda}\mathrm{P}^{-1}_{\lambda+1|\lambda}$ **and** $\mathrm{P}_{k+1|k}$: **Kalman prediction.**

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Block Matrix Representations of the ASD Covariance

$$
\mathbf{P}^k_{k:n} = \begin{pmatrix}
\mathbf{P}_{k|k} & \mathbf{P}_{k|k}\mathbf{W}^\top_{k-1|k} & \mathbf{P}_{k|k}\mathbf{W}^\top_{k-2|k} & \cdots & \mathbf{P}_{k|k}\mathbf{W}^\top_{1|k} \\
\mathbf{W}_{k-1|k}\mathbf{P}_{k|k} & \mathbf{P}_{k-1|k} & \mathbf{P}_{k-1|k}\mathbf{W}^\top_{k-2|k-1} & * & \mathbf{P}_{k-1|k}\mathbf{W}^\top_{1|k-1} \\
\mathbf{W}_{k-2|k}\mathbf{P}_{k|k} & \mathbf{W}_{k-2|k-1}\mathbf{P}_{k-1|k} & \mathbf{P}_{k-2|k} & * & \vdots \\
\vdots & * & * & * & \mathbf{P}_{2|k}\mathbf{W}^\top_{1|2} \\
\mathbf{W}_{1|k}\mathbf{P}_{k|k} & \mathbf{W}_{1|k-1}\mathbf{P}_{k-1|k} & \cdots & \mathbf{W}_{1|2}\mathbf{P}_{2|k} & \mathbf{P}_{1|k}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\mathbf{T}_{k|k} & -\mathbf{W}^\top_{k-1|k}\mathbf{Q}^{-1}_{k-1|k} & \mathbf{O} & \cdots & \mathbf{O} \\
-\mathbf{Q}^{-1}_{k-1|k}\mathbf{W}_{k-1|k} & \mathbf{T}_{k-1|k} & -\mathbf{W}^\top_{k-2|k}\mathbf{Q}^{-1}_{k-2|k} & \ddots & \vdots \\
\mathbf{O} & -\mathbf{Q}^{-1}_{k-2|k}\mathbf{W}_{k-2|k} & \ddots & \ddots & \mathbf{O} \\
\vdots & \ddots & \ddots & \mathbf{T}_{m+1|k} & -\mathbf{W}^\top_{m|k}\mathbf{Q}_{m|k} \\
\mathbf{O} & \cdots & \mathbf{O} & -\mathbf{Q}_{m|k}\mathbf{W}_{m|k} & \mathbf{T}_{m|k}
\end{pmatrix}^{-1}
$$

Markov property: tridiagonal structure of the inverse ASD covariance

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Closed-form Representations for ASDs

Example: conditions, where MHT tracking is appropriate

likelihood $\ell(\mathbf{x}_k; Z_k) \propto (1 - P_D)\rho_F + P_D \sum_{j_k=0}^{m_k} \mathcal{N}(\mathbf{z}_{j_k}; \mathbf{H}_k\mathbf{x}_k, \mathbf{R}_k)$

evolution model: $p(\mathbf{x}_k|\mathbf{x}_{k-1}) = \mathcal{N}(\mathbf{x}_k; \mathbf{F}_{k|k-1}\mathbf{x}_{k-1}, \mathbf{D}_{k|k-1})$

Induction argument (tedious, but elementary calculations):

The ASD is a Gaussian mixture:

$$p(\mathbf{x}_{k:n}|Z^k) = \sum_{\mathbf{j}_k} p_{k|k}^{\mathbf{j}_k} \, \mathcal{N}\left(\mathbf{x}_{k:n}; \mathbf{x}_{k:n}^{\mathbf{j}_k}, \mathbf{P}_{k:n}^{\mathbf{j}_k}\right)$$

with data interpretation *histories*

$$\mathbf{j}_k = (j_k, \ldots, j_1), \quad 0 \leq j_k \leq m_k.$$

Similar mixture ASDs result for IMM evolution models and IMM-MHT

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Application to OoS Measurement Processing

Latencies in the communication infrastructure: *Measurements can arrive at a processing node in a distributed data fusion system "too late", i.e. after sensor data with a time stamp newer than the time stamp of an out-of-sequence measurement have already been processed* [Seminal paper by Bar-Shalom, IEEE T-AES, 2002].

In situations where Kalman filtering is applicable,

let z_m be produced at time t_m with $n \leq m < k$, and be arriving at time t_k.

With a projection matrix Π_m, defined by $\Pi_m \mathrm{x}_{k:n} = \mathrm{x}_m$,

the impact of z_m on the present and past object states is described by:

$$\boxed{p(\mathrm{z}_m | \mathrm{x}_{k:n}) = \mathcal{N}(\mathrm{z}_m;\, \mathrm{H}_m \Pi_m \mathrm{x}_{k:n},\, \mathrm{R}_m).}$$

Direct generalizations to situations with ambiguous sensor data / IMM models.

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Standard Bayesian reasoning directly yields for the ASD:

$$p(\mathbf{x}_{k:n}|\mathbf{z}_m, Z^k) = \frac{p(\mathbf{z}_m|\mathbf{x}_{k:n})\, p(\mathbf{x}_{k:n}|Z^k)}{\int \mathrm{d}\mathbf{x}_{k:n}\, p(\mathbf{z}_m|\mathbf{x}_{k:n})\, p(\mathbf{x}_{k:n}|Z^k)} = \mathcal{N}\Big(\mathbf{x}_{k:n};\, \mathbf{x}_{k:m:n},\, \mathbf{P}_{k:m:n}\Big)$$

with parameters obtained by a version of the Kalman update:

$$\mathbf{x}_{k:m:n} = \mathbf{x}_{k:n} + \mathbf{W}_{k:m:n}(\mathbf{z}_m - \mathbf{H}_m \mathbf{\Pi}_m \mathbf{x}_{k:n})$$
$$\mathbf{P}_{k:m:n} = \mathbf{P}_{k:n} - \mathbf{W}_{k:M:n}\mathbf{S}_{k,m,1}\mathbf{W}_{k:m:n}^\top$$

$$\mathbf{S}_{k:m:n} = \mathbf{H}_m \mathbf{\Pi}_m \mathbf{P}_{k:n} \mathbf{\Pi}_m^\top \mathbf{H}_m^\top + \mathbf{R}_m$$
$$\mathbf{W}_{k:m:n} = \mathbf{P}_{k:n} \mathbf{\Pi}_m^\top \mathbf{H}_m^\top \mathbf{S}_{k:m:n}^{-1}.$$

Note that $\mathbf{S}_{k:m:n}$ to be inverted has the same dimension as $\mathbf{z}_m$.

Simultaneous update step for filtering and retrodiction

Efficient realizations exploit the structure of the ASD covariance.

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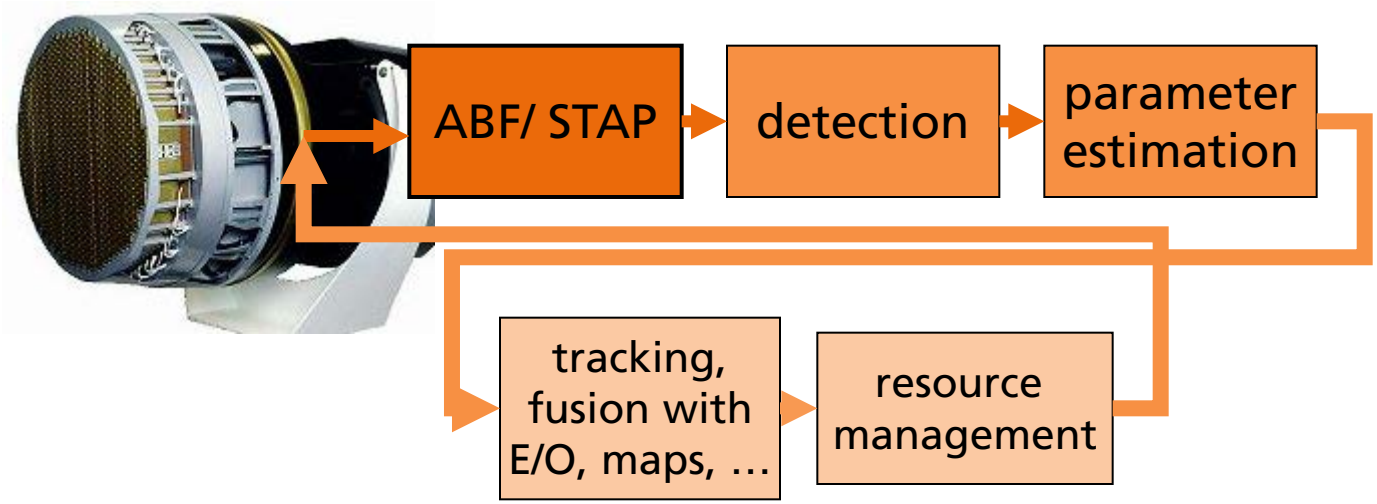
1. Multiple Hypothesis Tracking: Basic Principles

2. Tracking & Management for Phased Array Radar

3. GMTI Tracking and Fusion of Context Information

Sensor Management: Use efficiently the potential of multichannel systems!

signal processing



data fusion

multichannel system

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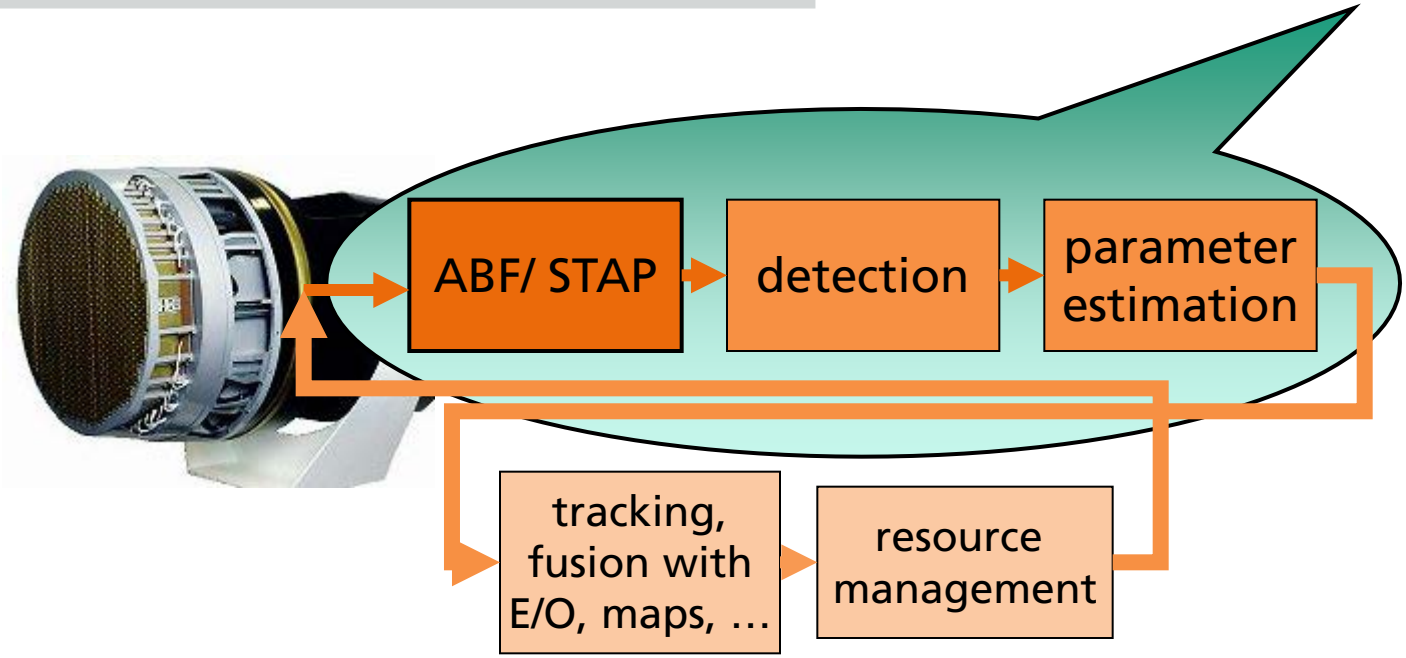
Sensor Management: Use efficiently the potential of multichannel systems!



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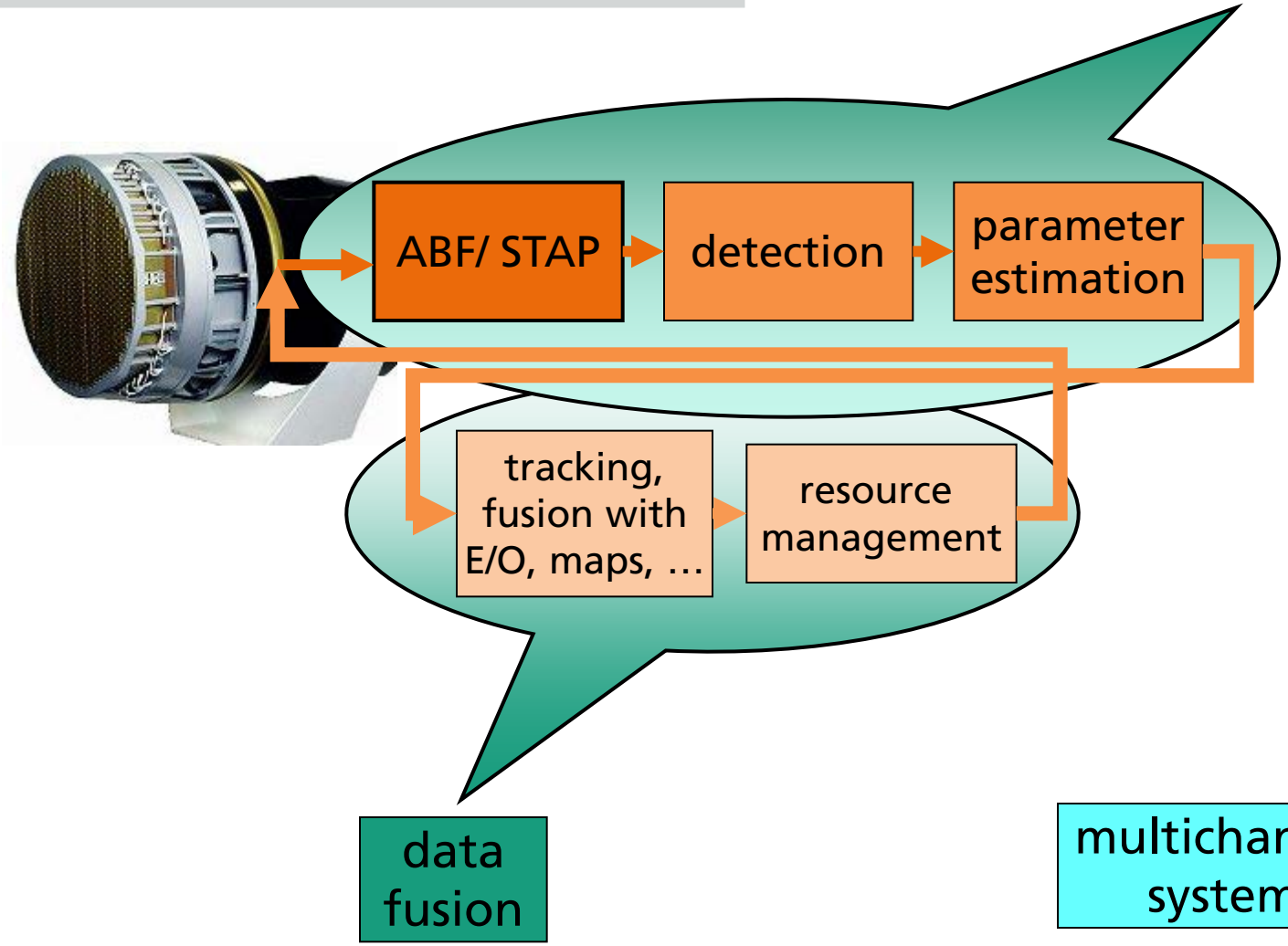
Sensor Management: Use efficiently the potential of multichannel systems!



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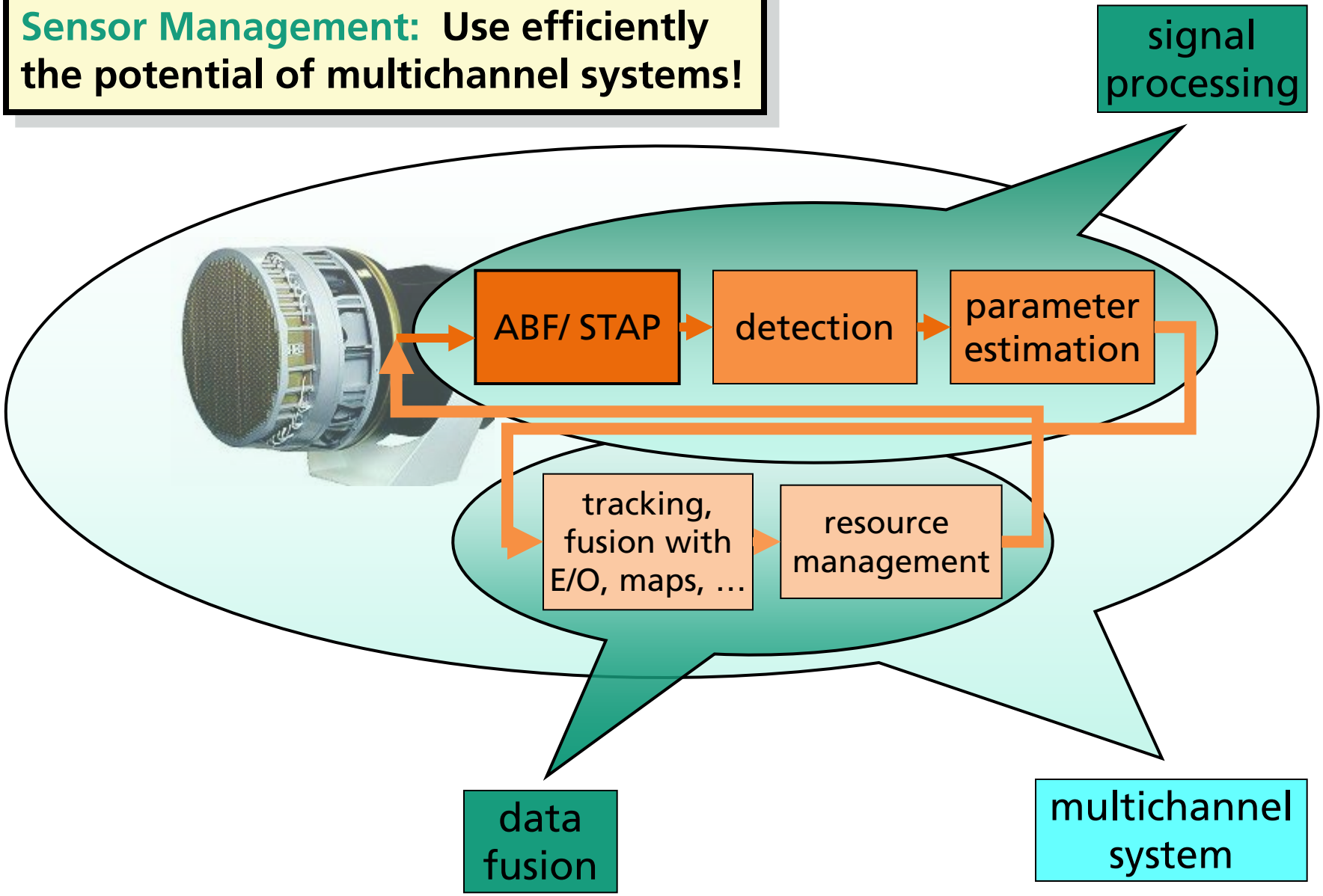

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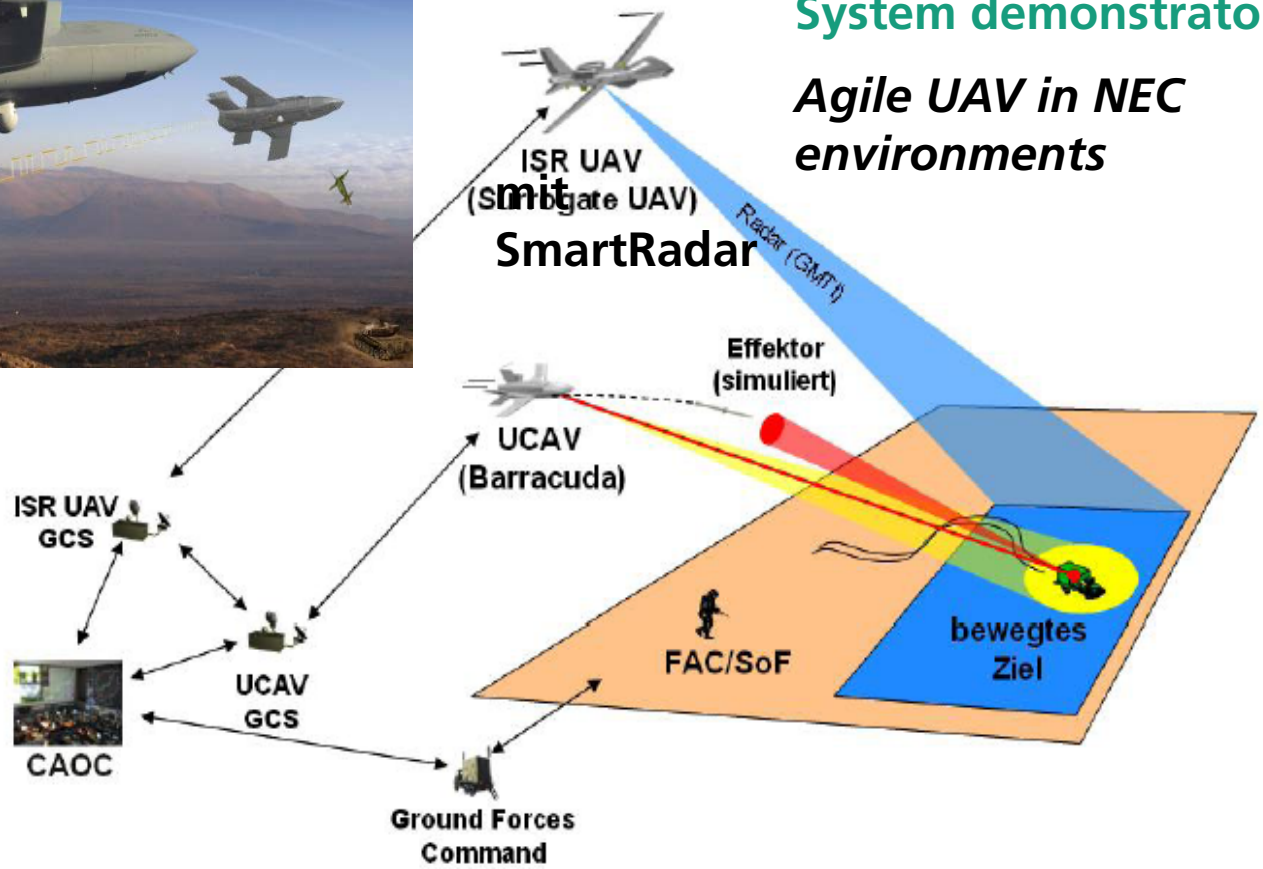
System demonstrator

Agile UAV in NEC environments



Contributions FKIE

- Adaptive GMTI-Tracking
- Dynamic sensor cueing
- Fusion GMTI + E/O sensors
- Using road maps
- Experimental evaluation

with
CASSIDIAN Air Systems

Summer 2012 (Goose Bay, CAN)

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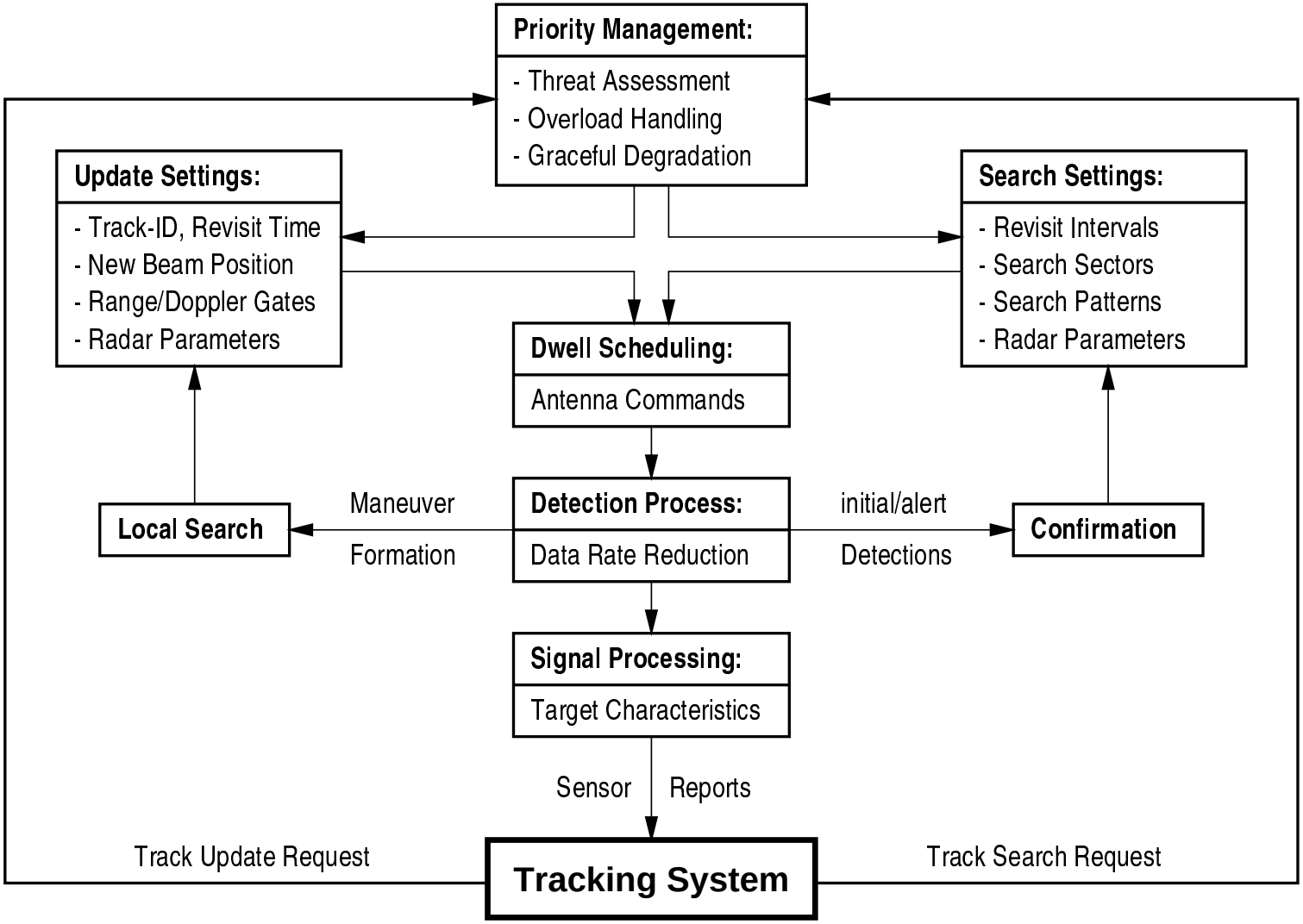
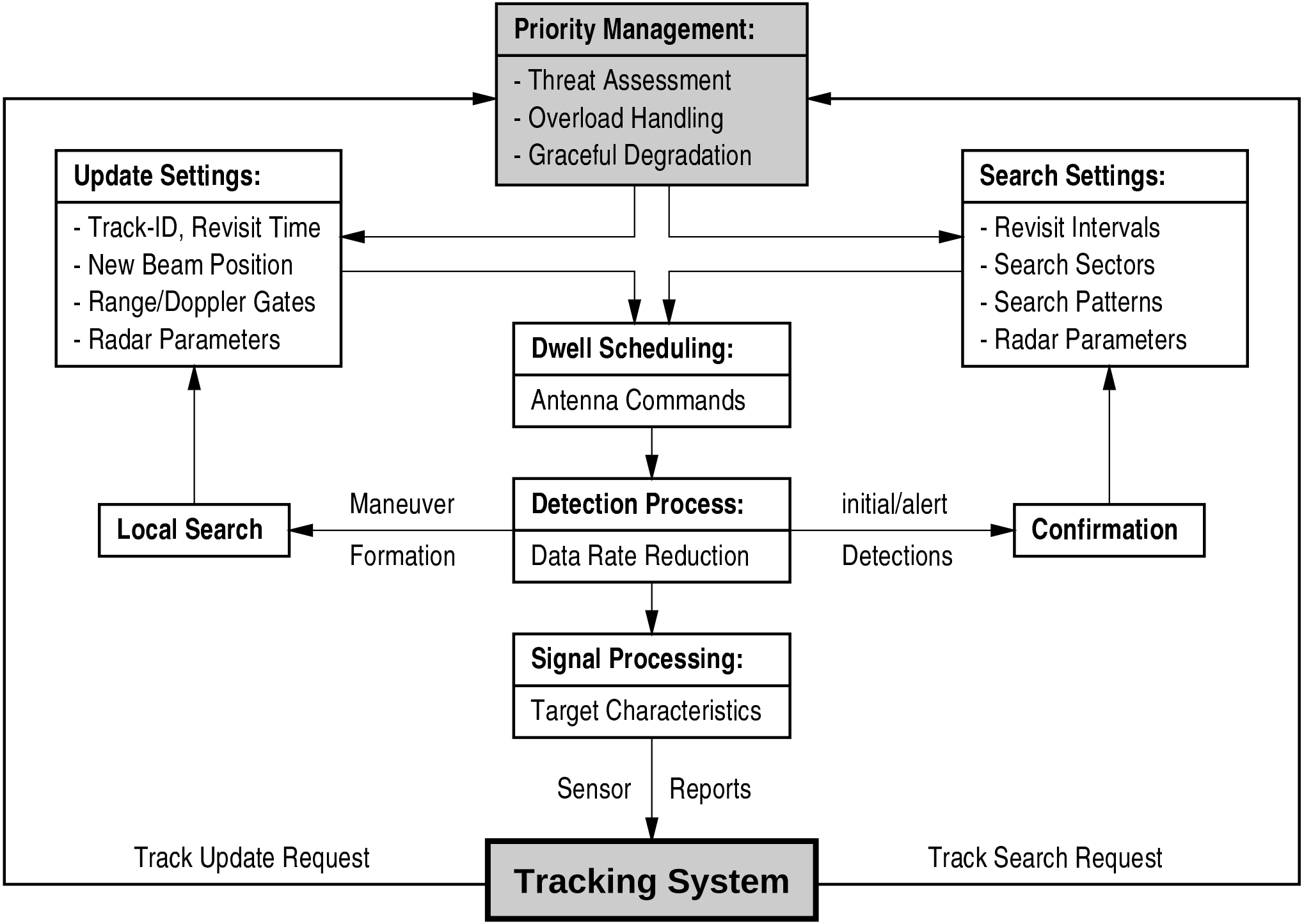
Radar Function Control: Information Flow





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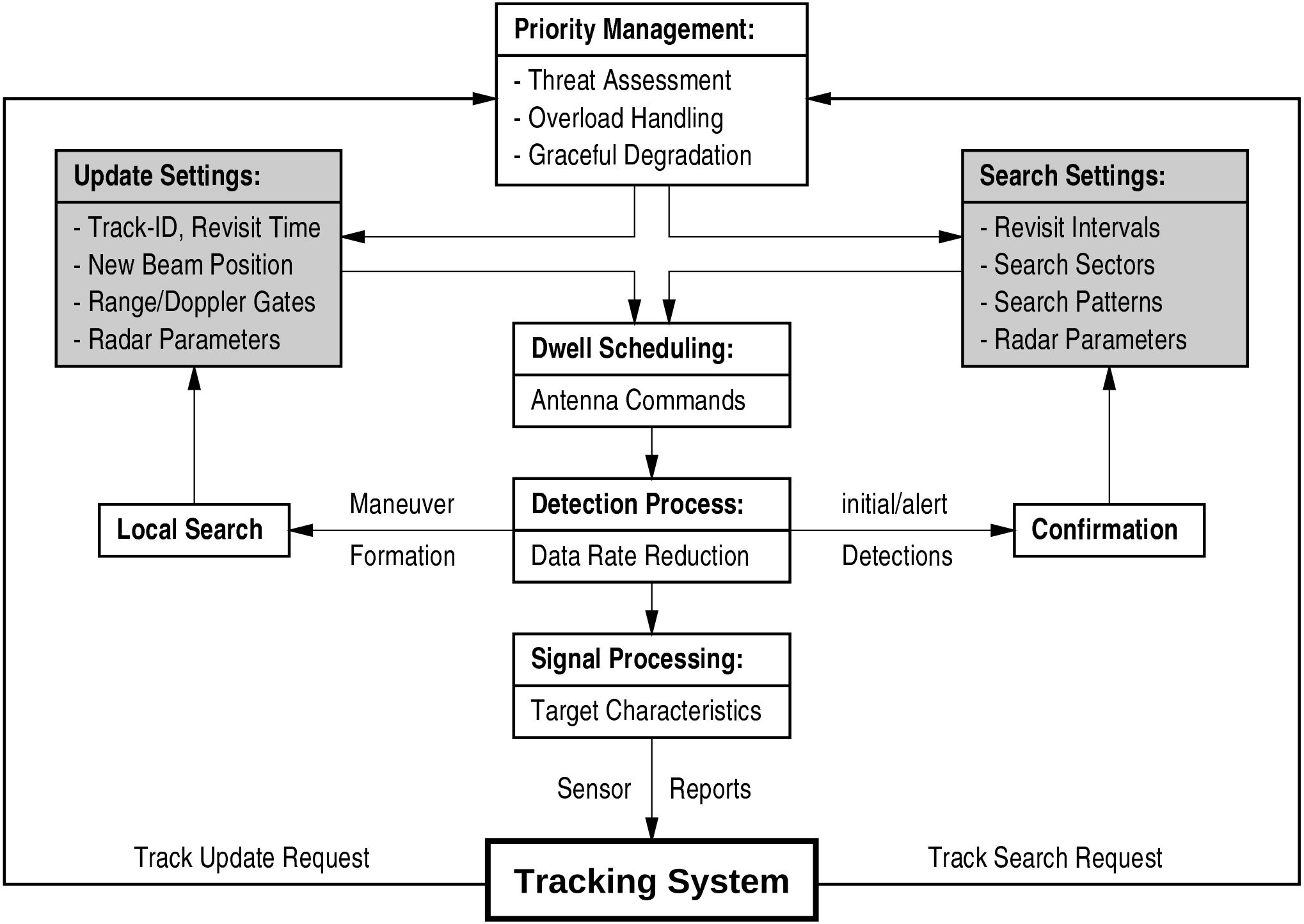

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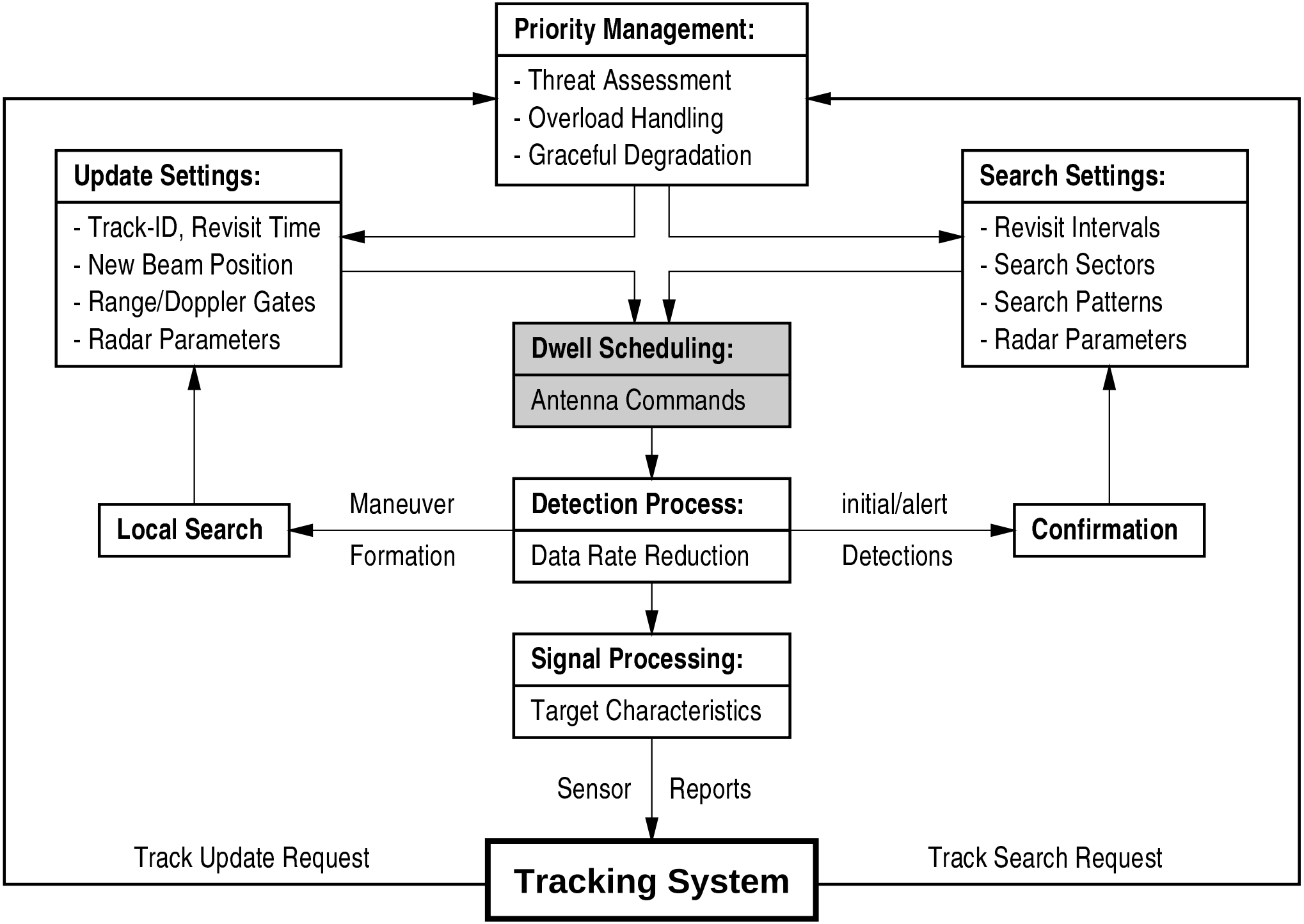

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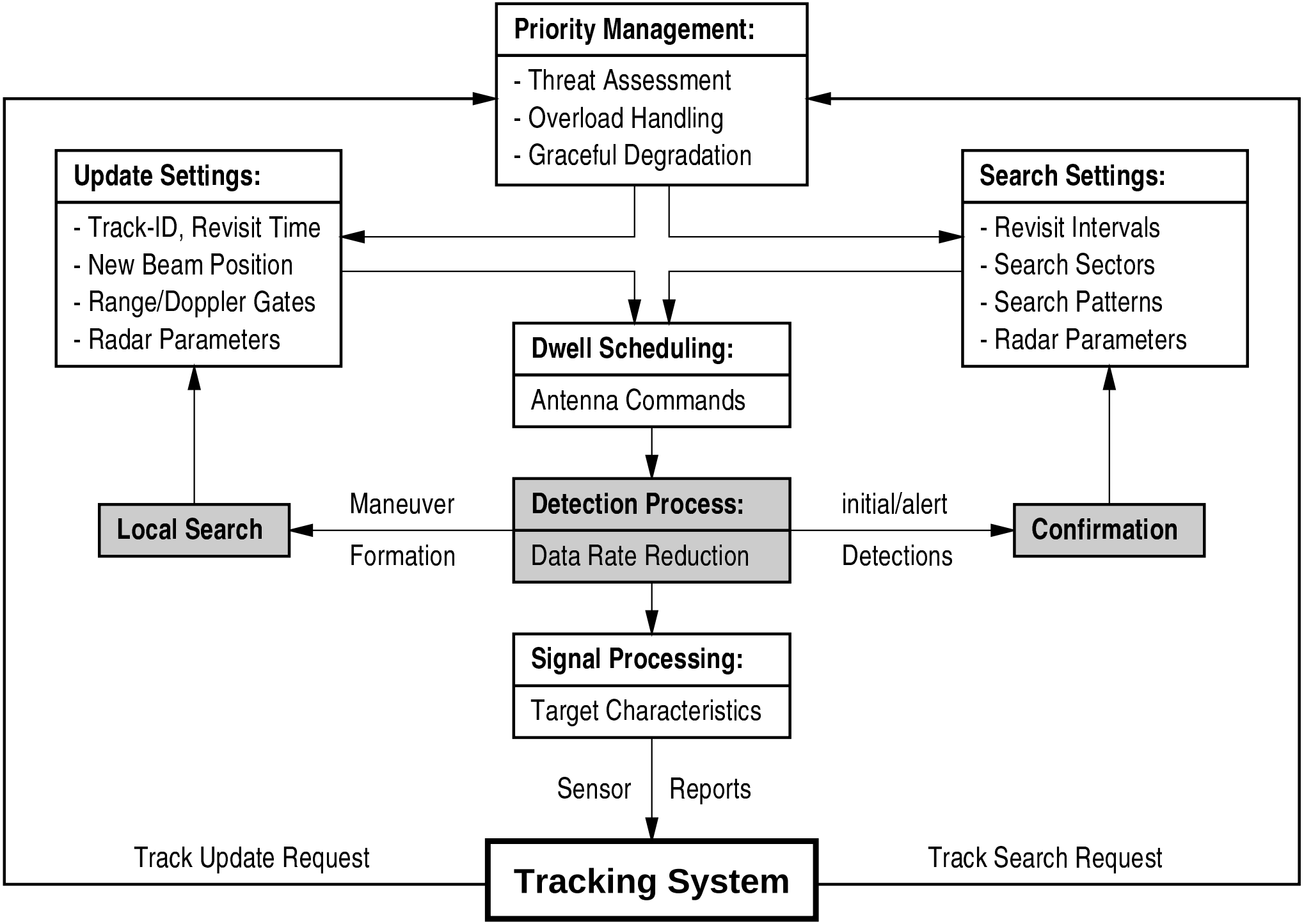
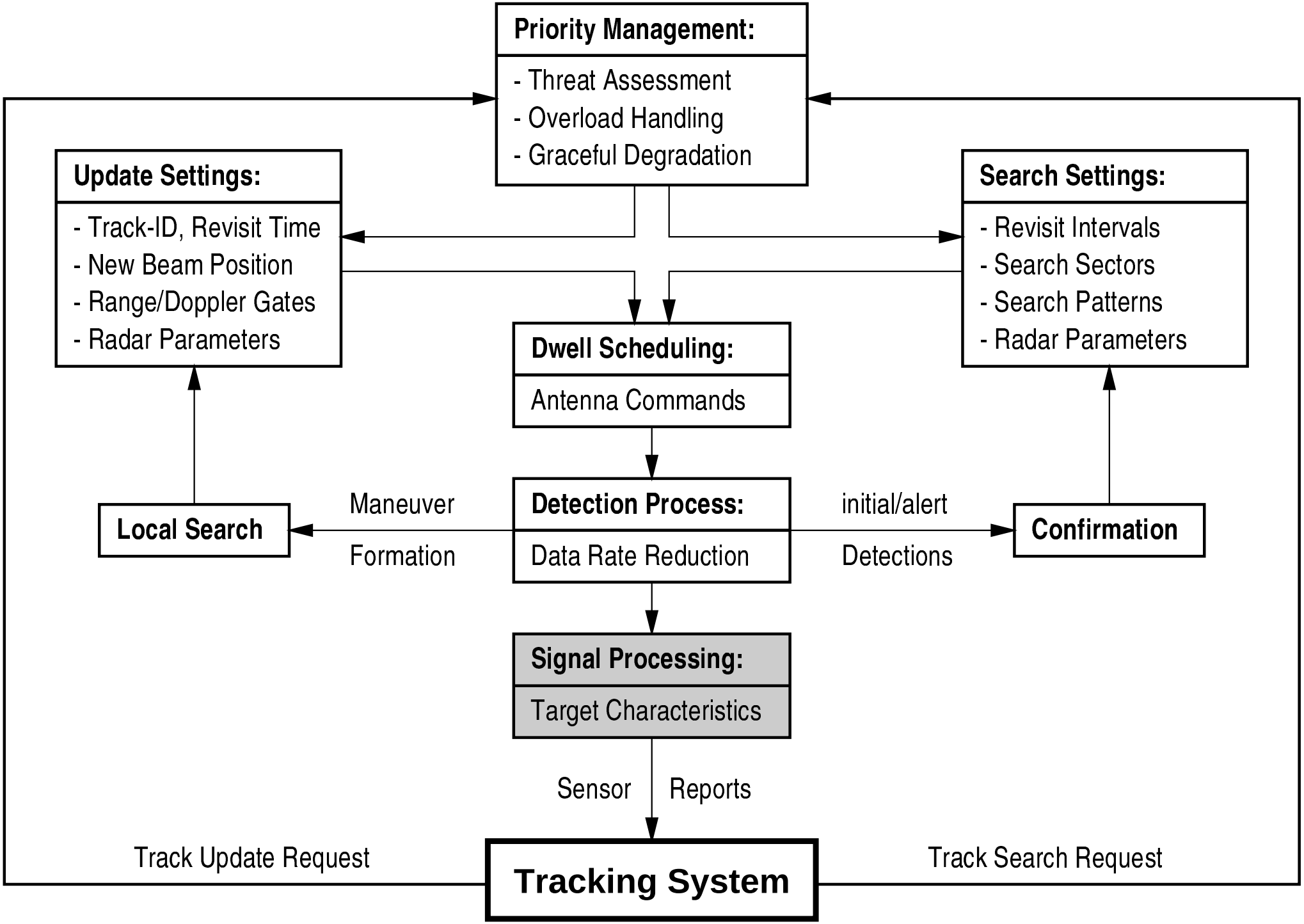


Fraunhofer FKIE



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Phased Array Radar: Adaptive Local Search

sensor resources allocated by the tracker

- adaptive revisit time control (lack of information)
- selective radar beam positioning (state prediction)
- if necessary: locally optimal search ($\rightarrow$ BAYES)

a simple radar beam and detection model

- beam positioning error: $\Delta \mathbf{b}_k^2 = |\mathbf{x}_k - \mathbf{b}_k|^2 / B^2$

  $\mathbf{x}_k$: target direction, $\mathbf{b}_k$: current beam position, B: radar beam width

- signal-to-noise ratio: $s(\mathbf{x}_k; \mathbf{b}_k) = s_0 \, \mathrm{e}^{-\log 2 \Delta \mathbf{b}_k^2}$
- detection probability: $P_D(\mathbf{x}_k; \mathbf{b}_k) = P_F^{\frac{1}{1+s(\mathbf{x}_k;\mathbf{b}_k)}}$

BAYESian Beam Positioning and Local Search

1. first dwell $\rightarrow$ point radar beam to predicted target position $\rightarrow \mathbf{b}_k^1$
2. successful detection $\rightarrow$ filtering/prediction/revisit time $\rightarrow$ GOTO 1!

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3. no success $\rightarrow$ Bayesian processing of this 'negative' output ($\neg \mathsf{D}_1$):

$$\boxed{p(\mathbf{x}_k|\neg \mathsf{D}_1, \mathcal{Z}^{k-1}) \propto \left[1 - P_D(\mathbf{x}_k; \mathbf{b}_k^1)\right] p(\mathbf{x}_k|\mathcal{Z}^{k-1})}$$

4. next dwell $\rightarrow$ point beam to the maximum of this conditional pdf!

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1. first dwell $\rightarrow$ point radar beam to predicted target position $\rightarrow \mathbf{b}_k^1$
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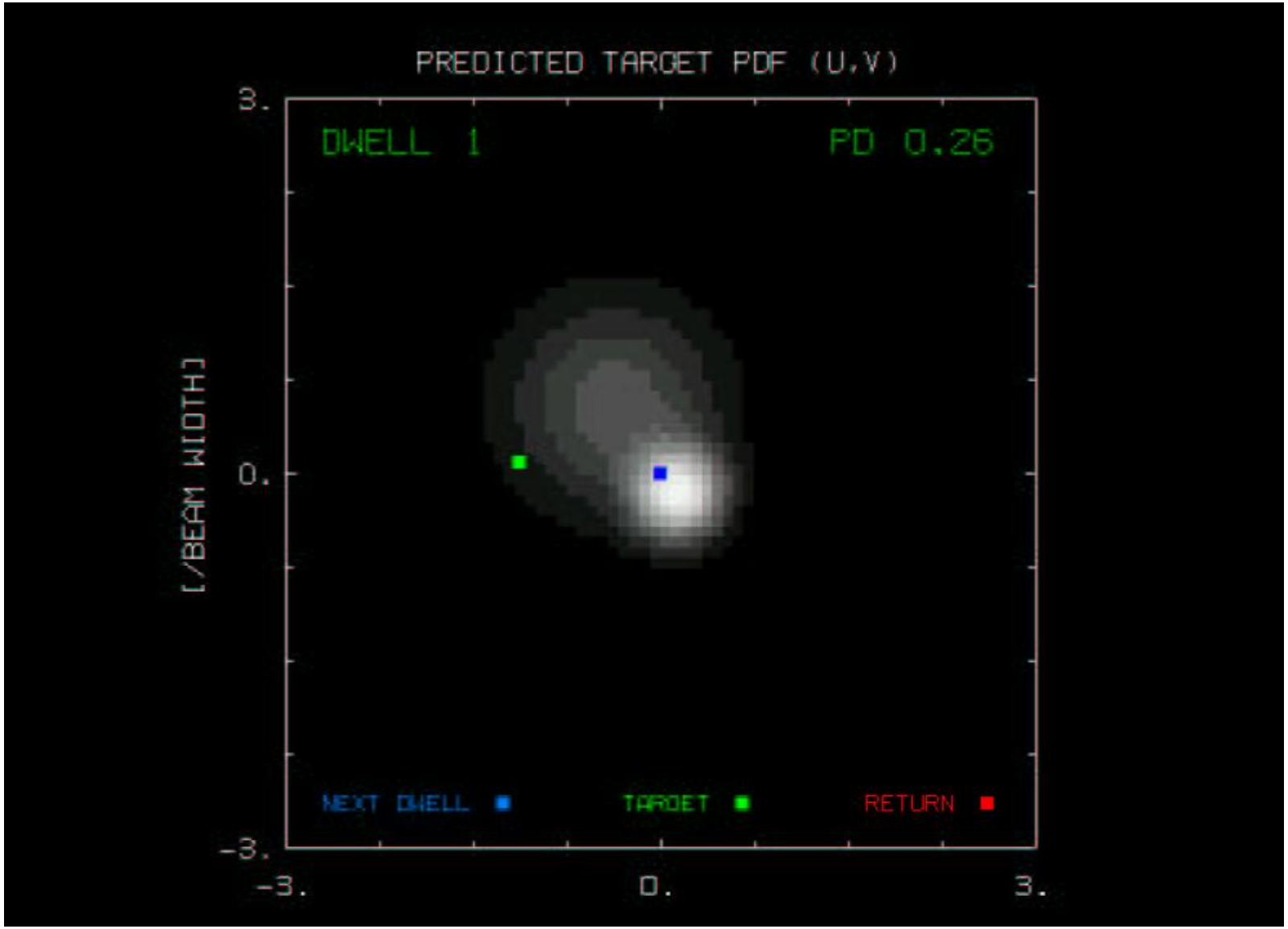
4. next dwell $\rightarrow$ point beam to the maximum of this conditional pdf!
5. successful detection $\rightarrow$ filtering/prediction/revisit time $\rightarrow$ GOTO 1!
6. no success $\rightarrow$ calculate: $p(u_k, v_k|\neg \mathsf{D}_1, \neg \mathsf{D}_2, \mathcal{Z}^{k-1})\rightarrow$ GOTO 4!

BAYESian local search: example with an IMM pdf



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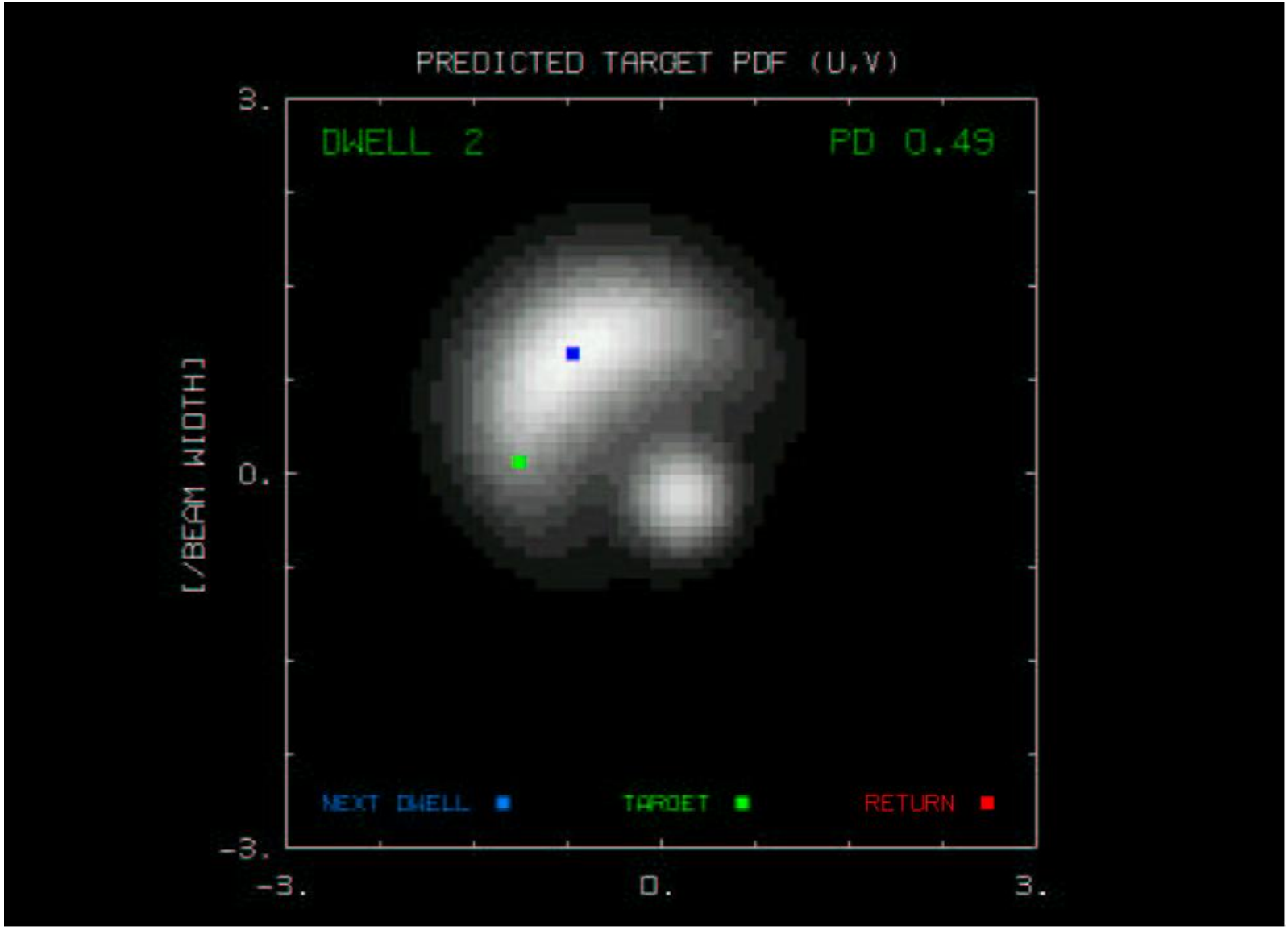

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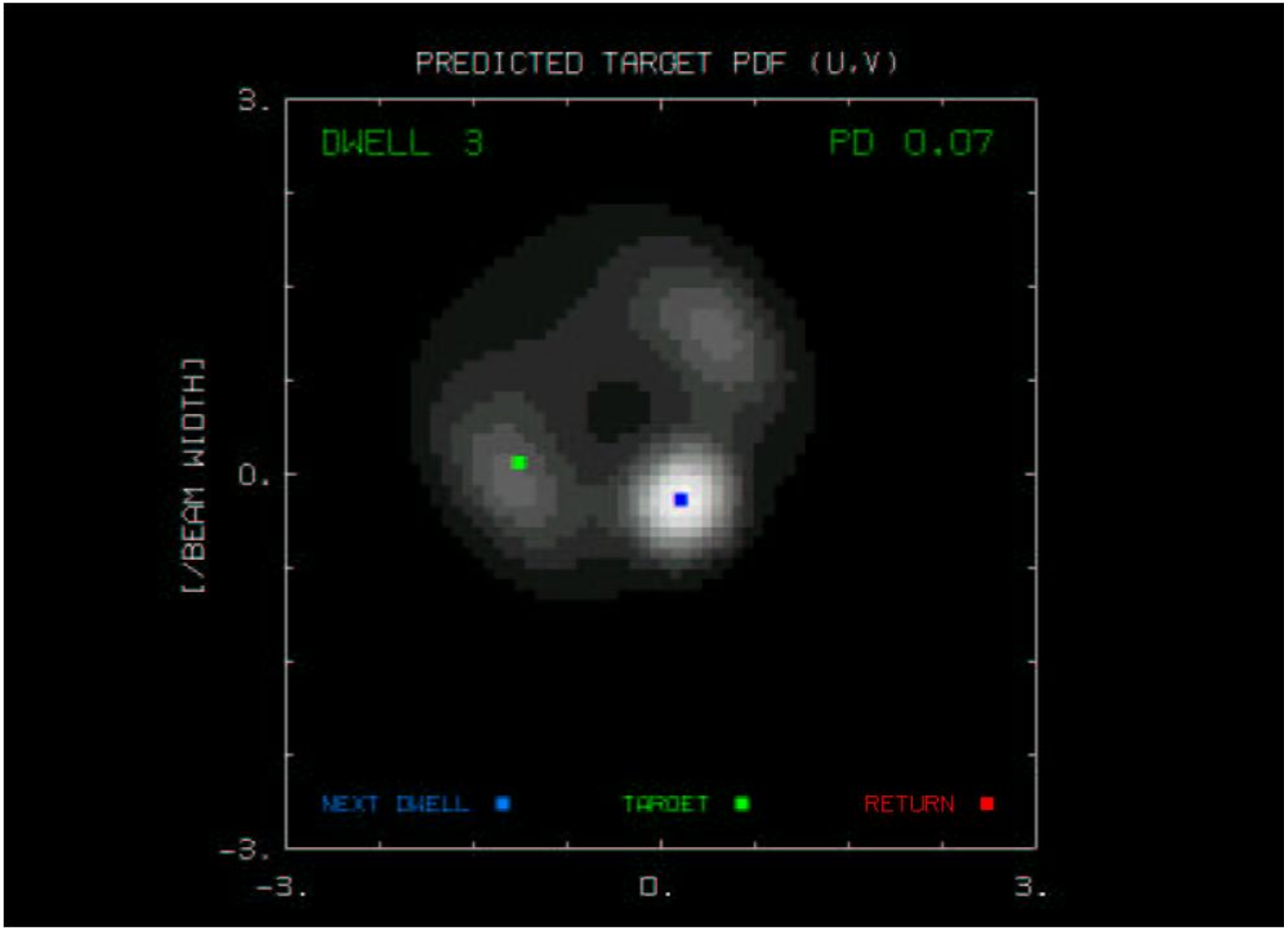

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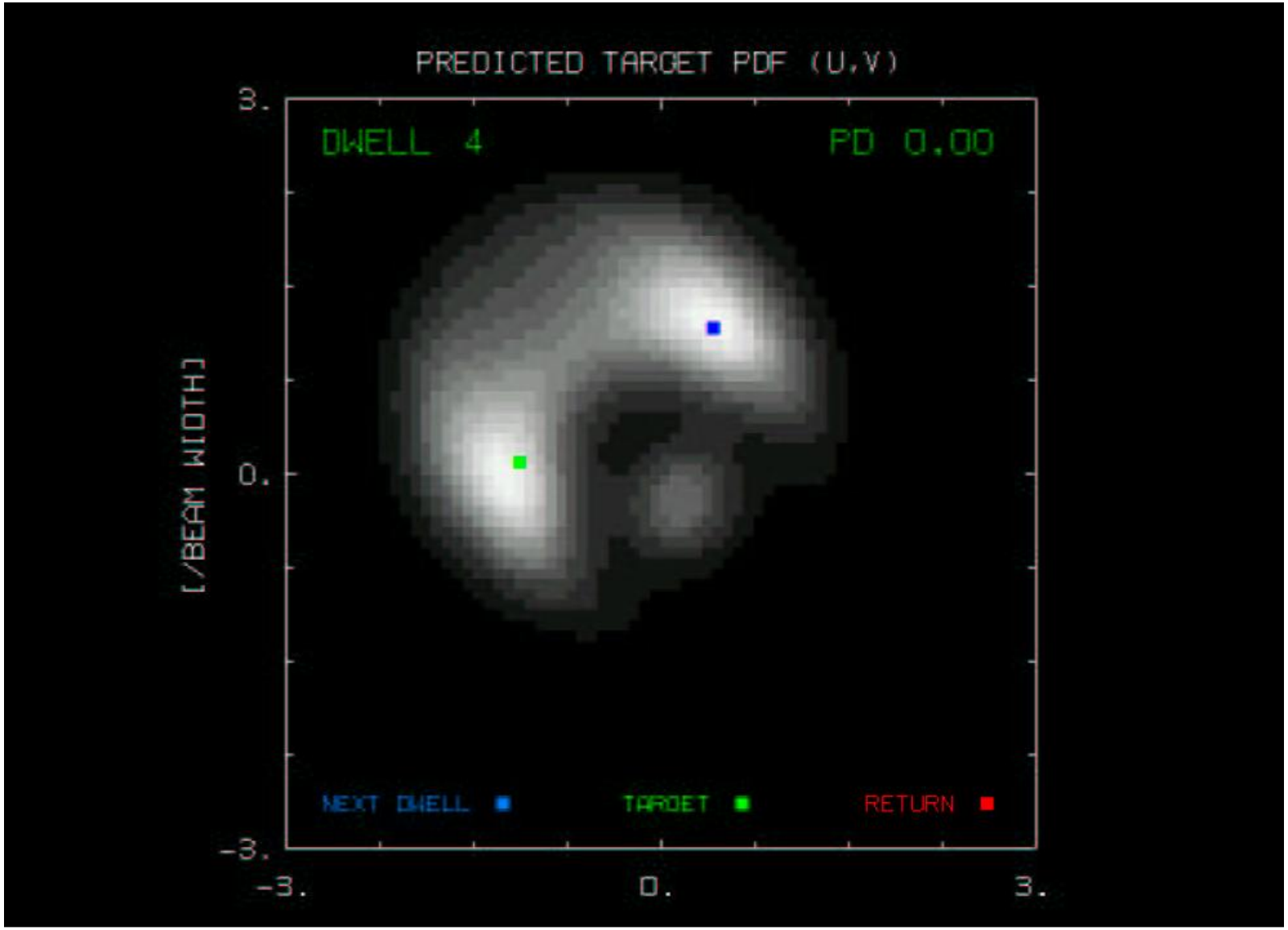

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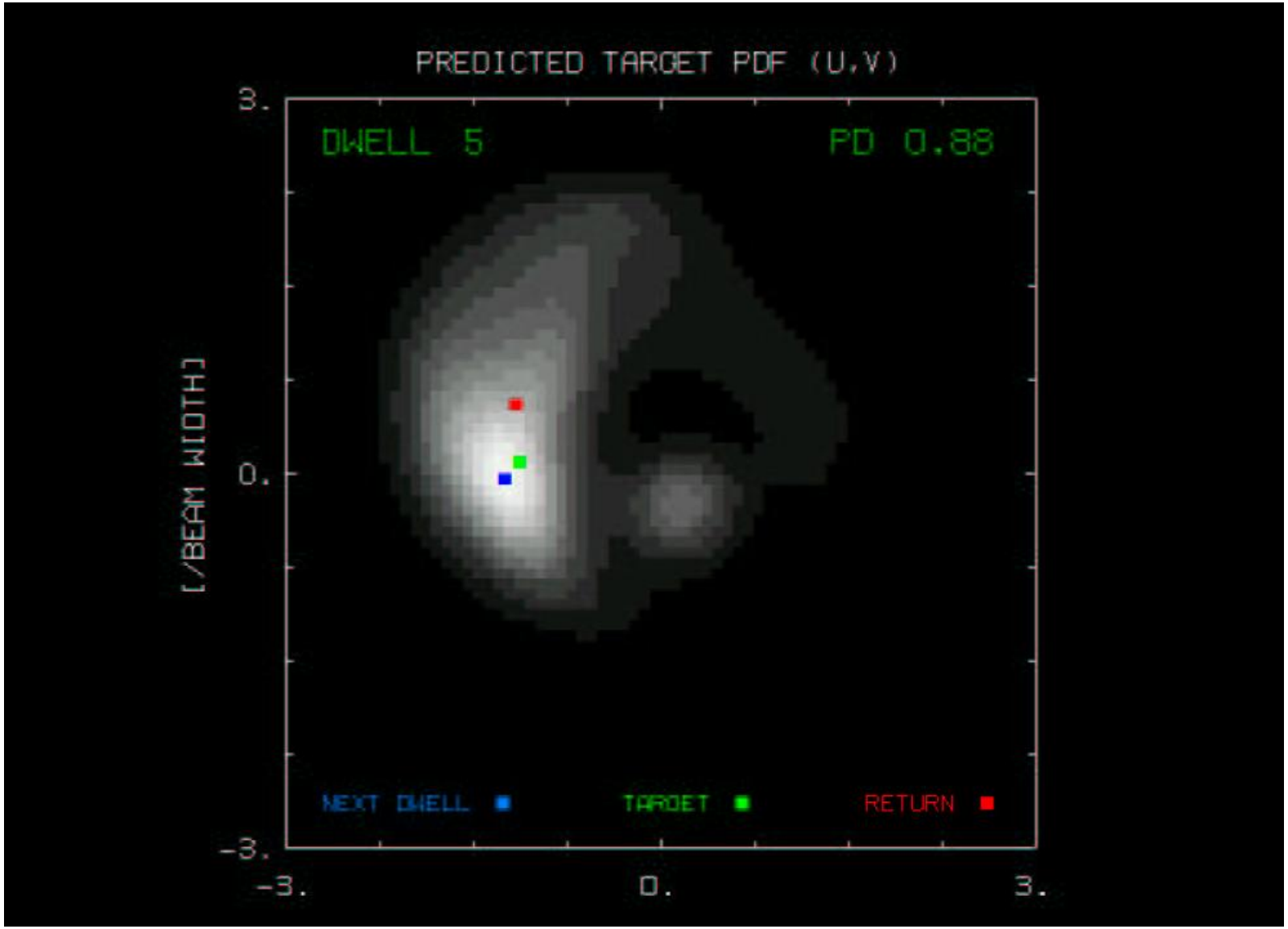

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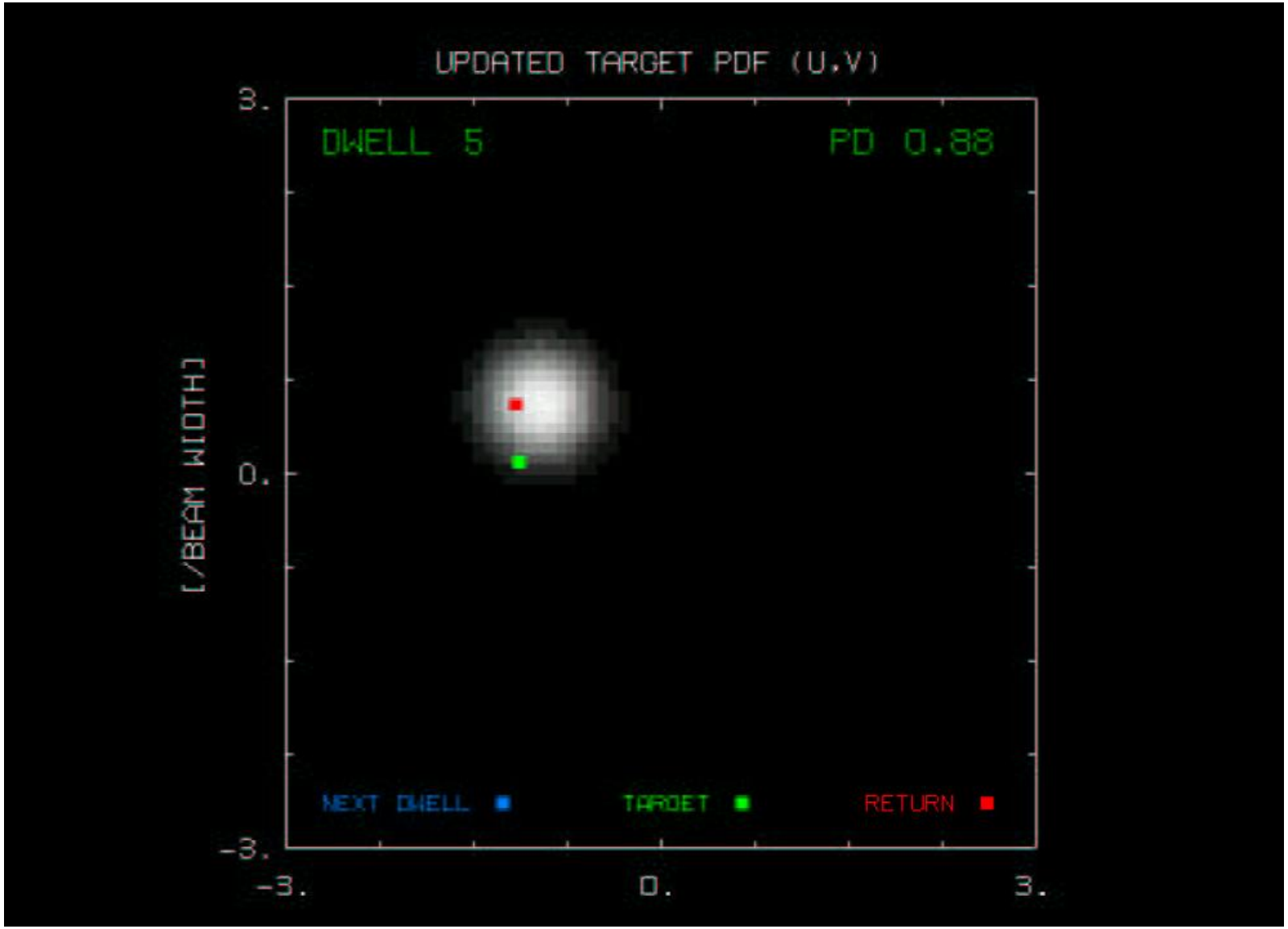


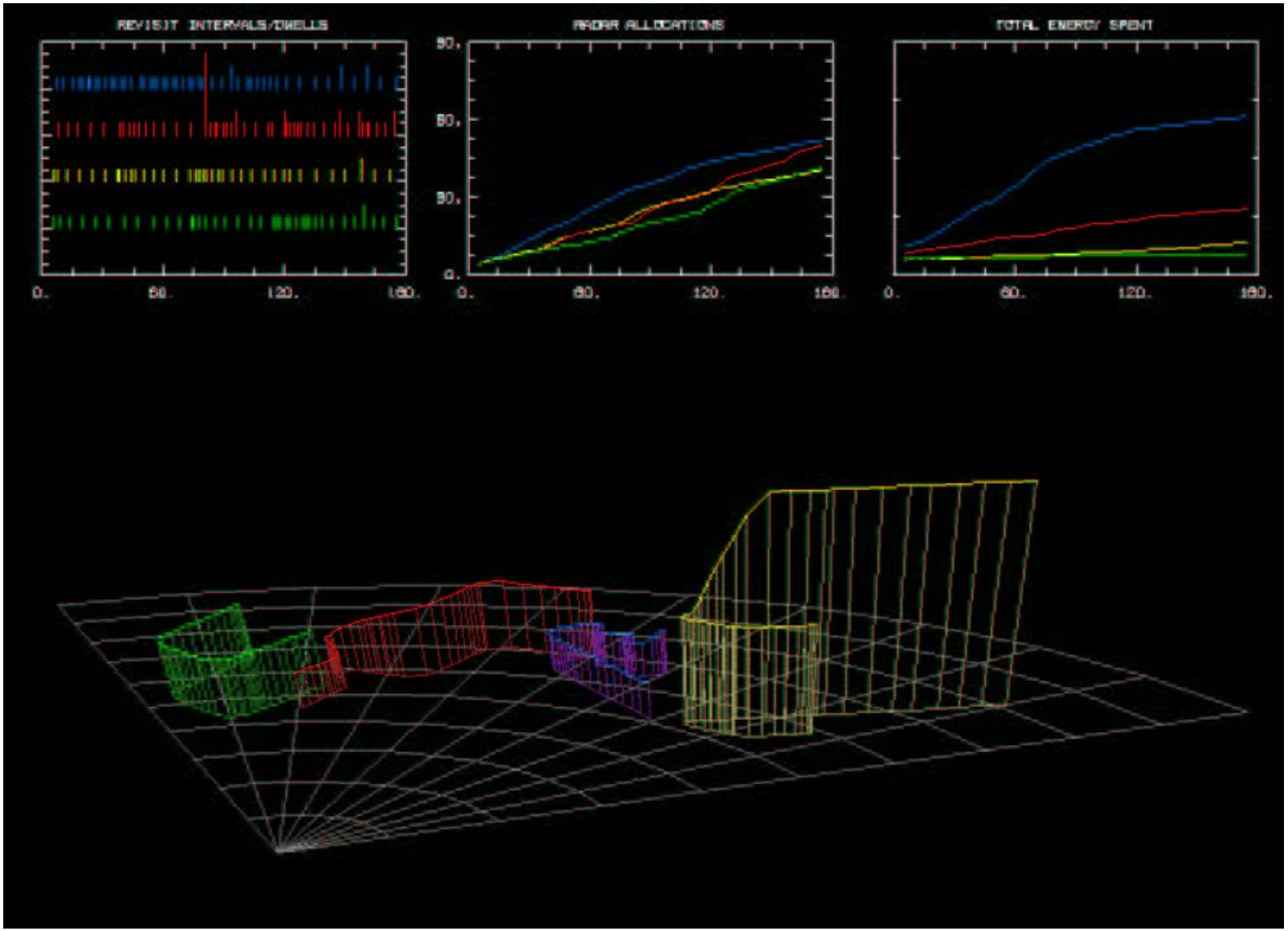
A *'negative' sensor output* - i.e. missing, but expected data - provides information on the target position.

- **direct impact on sensor management**
- **prerequisite: appropriate sensor model**

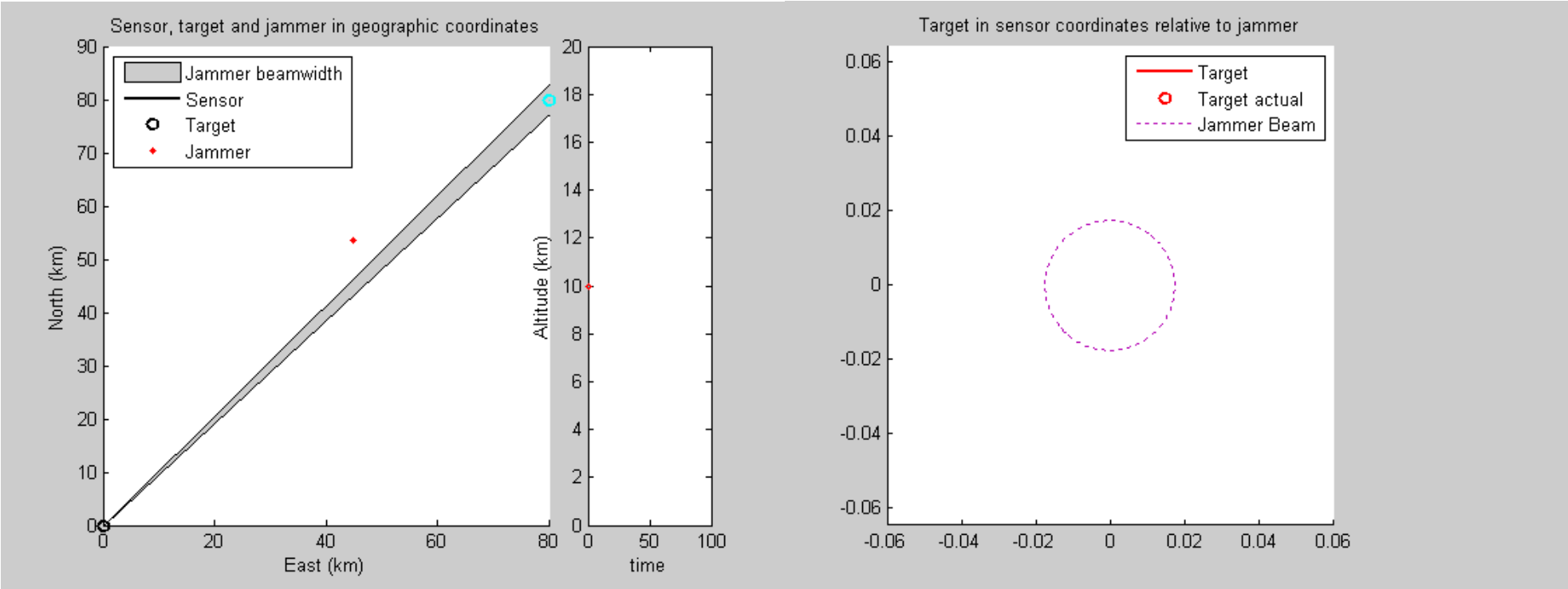
Koch, W., Advanced Sensor and Dynamics Models with an Application to Sensor Management. *Chapter 1 in: Nada Milisavljevic (Ed.), Sensor and Data Fusion, IN-TECH Publishers, February 2009*.

Koch, W., On Exploiting Negative Sensor Evidence for Target Tracking and Sensor Data Fusion. *International Journal on Information Fusion, Volume 8, Issue 1, p.28-39, Elsevier, January 2007*.

Phased-Array Tracking: Adaptive Sensor Management



Adaptive Tracking and Resource Management for ECCM



3D, jammer on a circle, sensor: sinusoidal, target trying to hide in jammer notch

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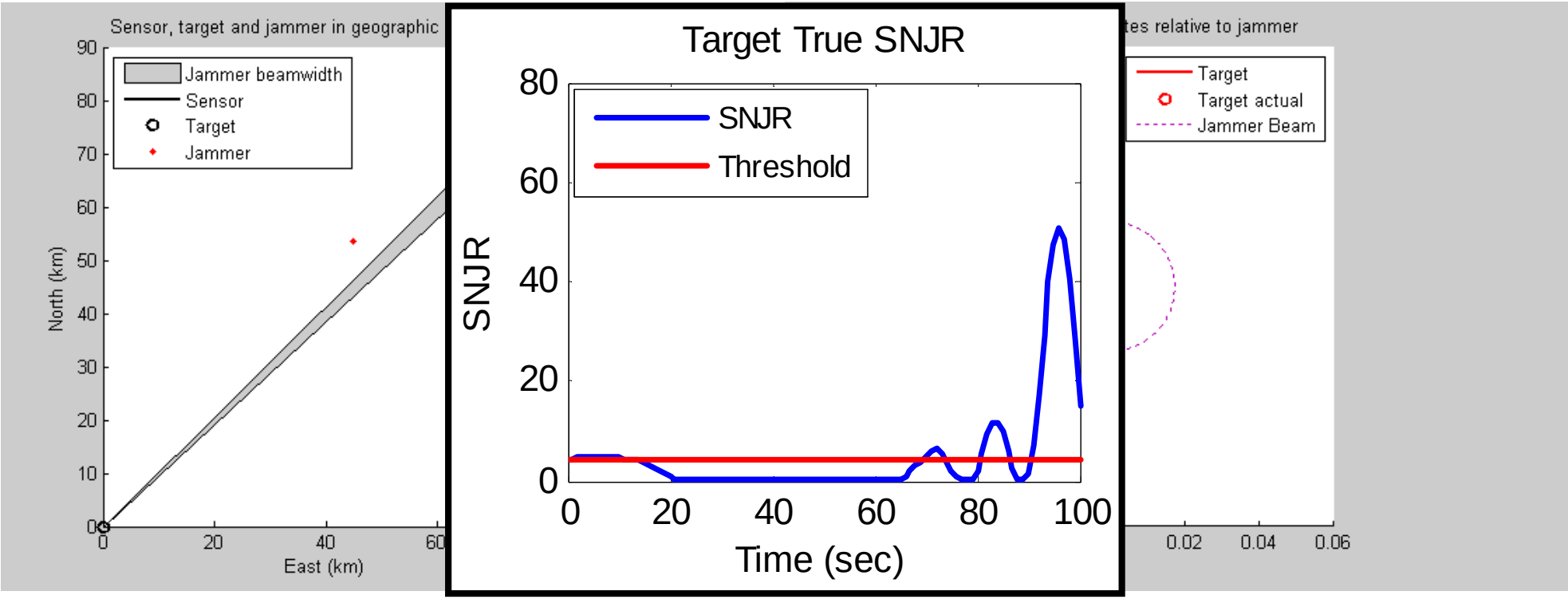
Adaptive Tracking and Resource Management for ECCM



3D, jammer on a circle, sensor: sinusoidal, target trying to hide in jammer notch

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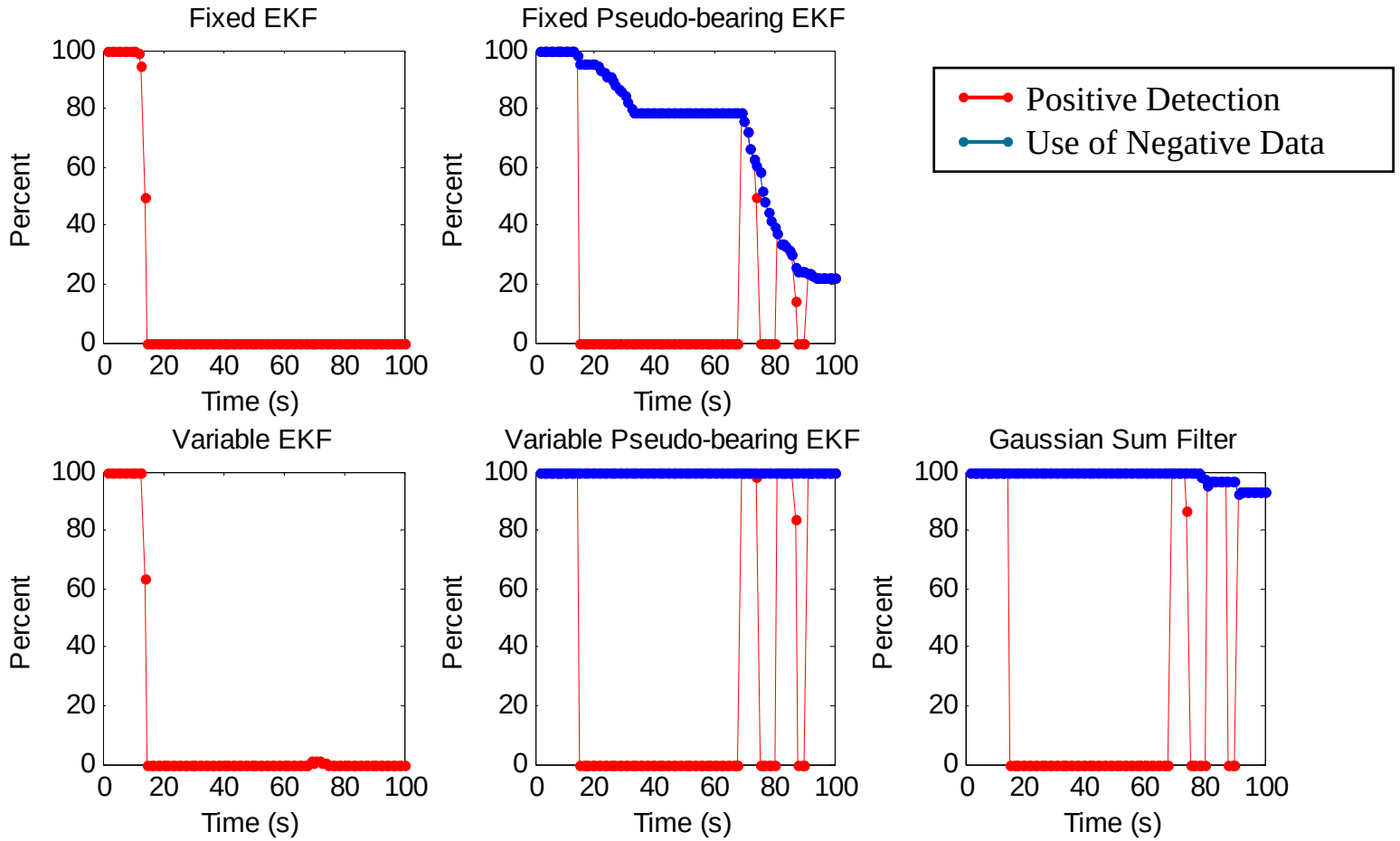
Adaptive Tracking and Resource Management for ECCM

Effect on Track Continuity

Proportion of dwells producing a real or pseudo measurement (250 Monte Carlo runs)



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Adaptive Tracking and Resource Management for ECCM

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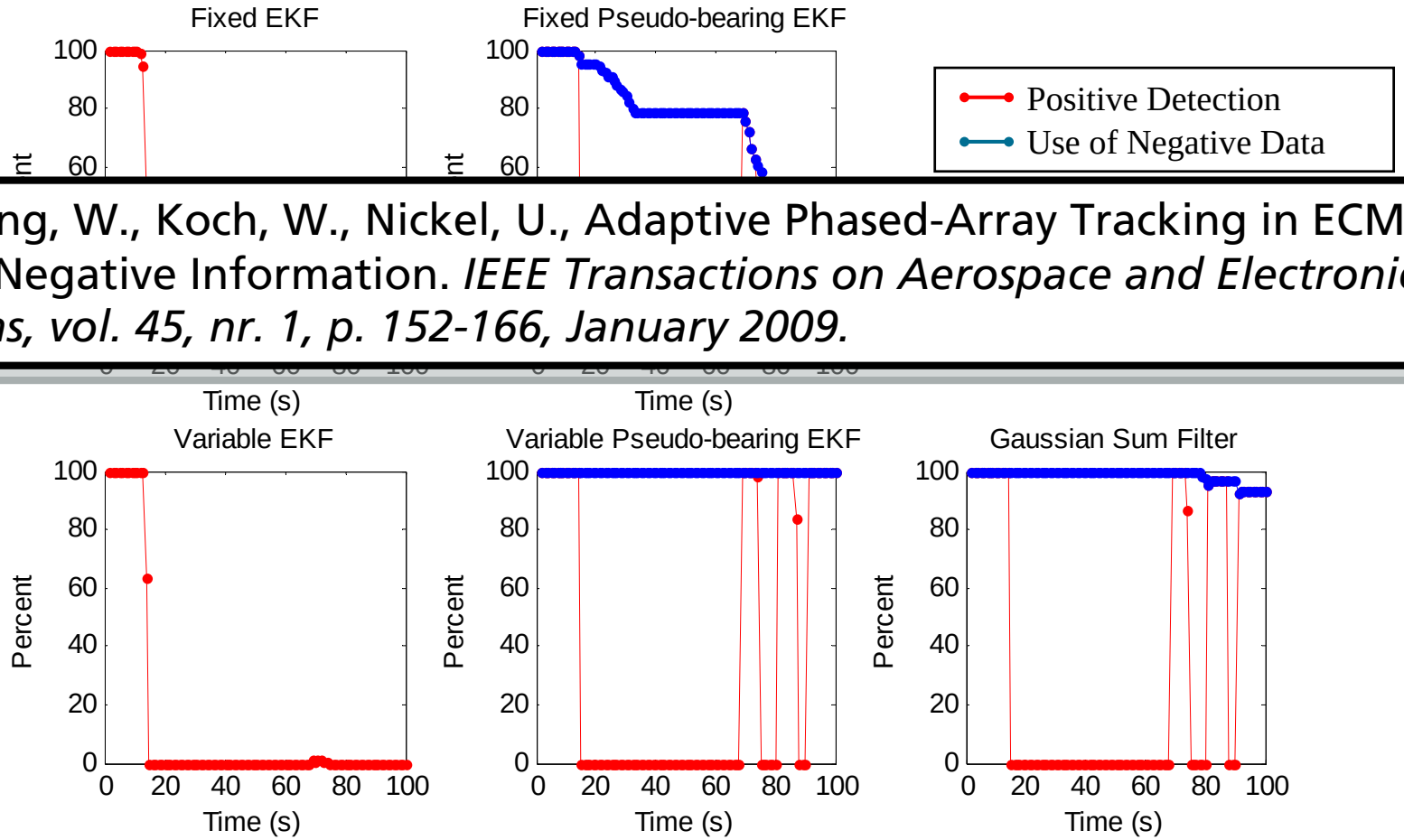

Blanding, W., Koch, W., Nickel, U., Adaptive Phased-Array Tracking in ECM Using Negative Information. *IEEE Transactions on Aerospace and Electronic Systems, vol. 45, nr. 1, p. 152-166, January 2009.*

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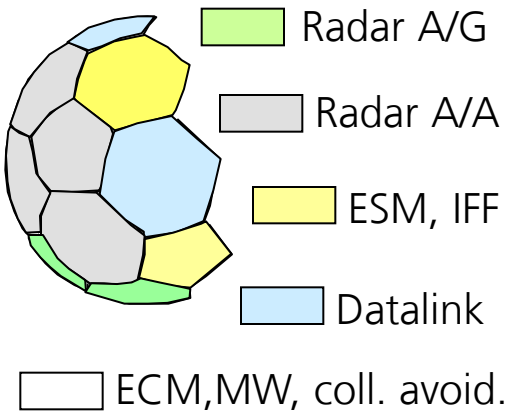
Current Research for Future Situational Awareness, Self-protection & Effectiveness:

Airborne Integrated Multifunctional RF Sensor



- **High operational impact in a network centric context**
  fighter aircraft / Unmanned Combat Aerial Vehicles (UCAV)
  application-driven R&D with direct industrial participation
- **Seamless integration of heterogeneous RF functions**
  various radar modes, electronic support/counter measures, missile warning, secure/jam-resistant comms & SatNAV, interface IR- E/O
- **Challenging research questions opening new horizons**
  advanced adaptive tracking technology, heterogeneous sensor data fusion, mode-selective resource management

Interesting research questions regarding the entire processing chain (i.e. all JDL levels):

advanced algorithms, simulation: performance, evaluation & prediction, proof-of-concept, design recommendations

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1. Multiple Hypothesis Tracking: Basic Principles

2. Tracking & Management for Phased Array Radar

3. GMTI Tracking and Fusion of Context Information

Tracking Application: Ground Picture Production

GMTI Radar: Ground Moving Target Indicator

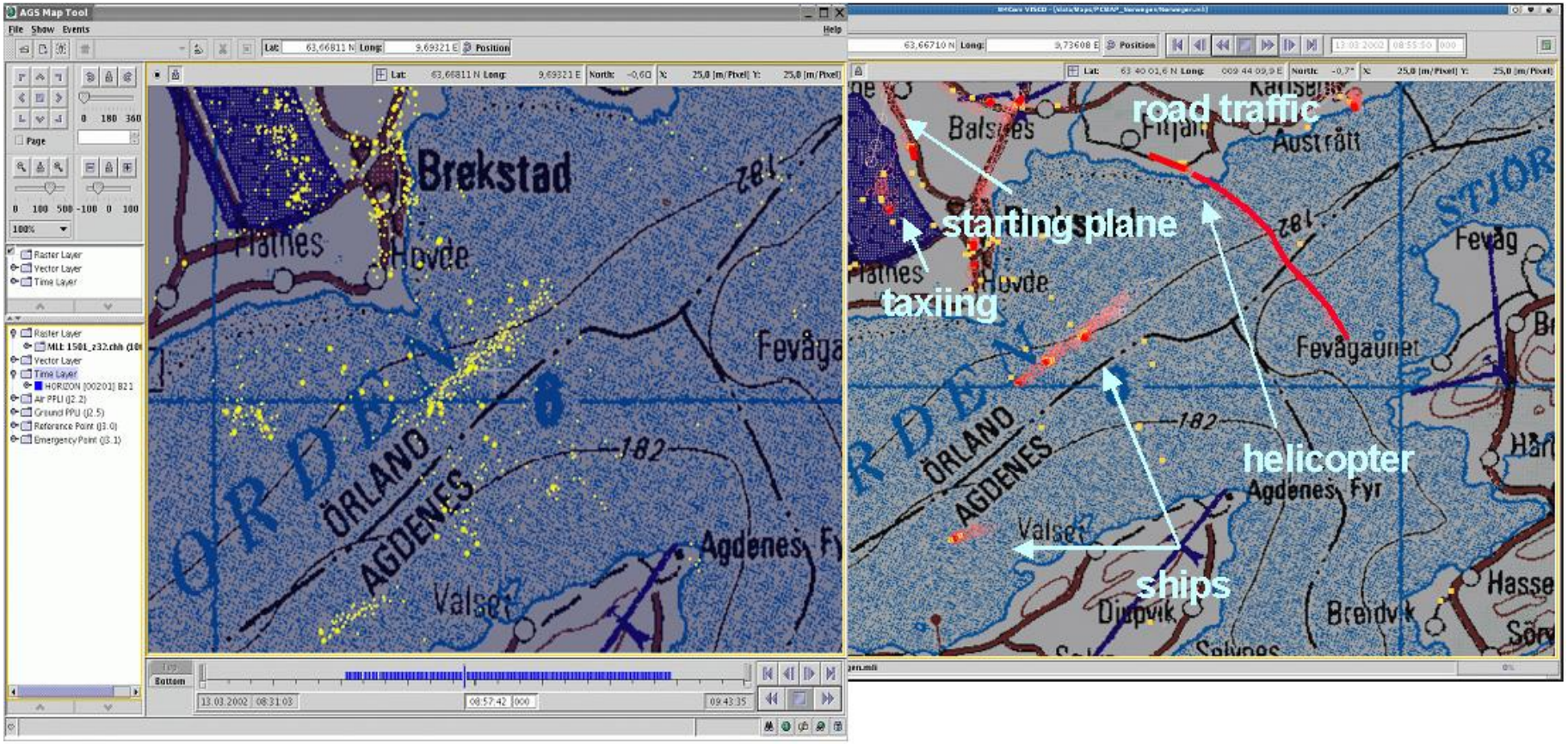
wide area, all-weather, day/night, real-time surveillance of
a dynamically evolving ground or near-to-ground situation

GMTI Tracking: Some Characteristic Aspects

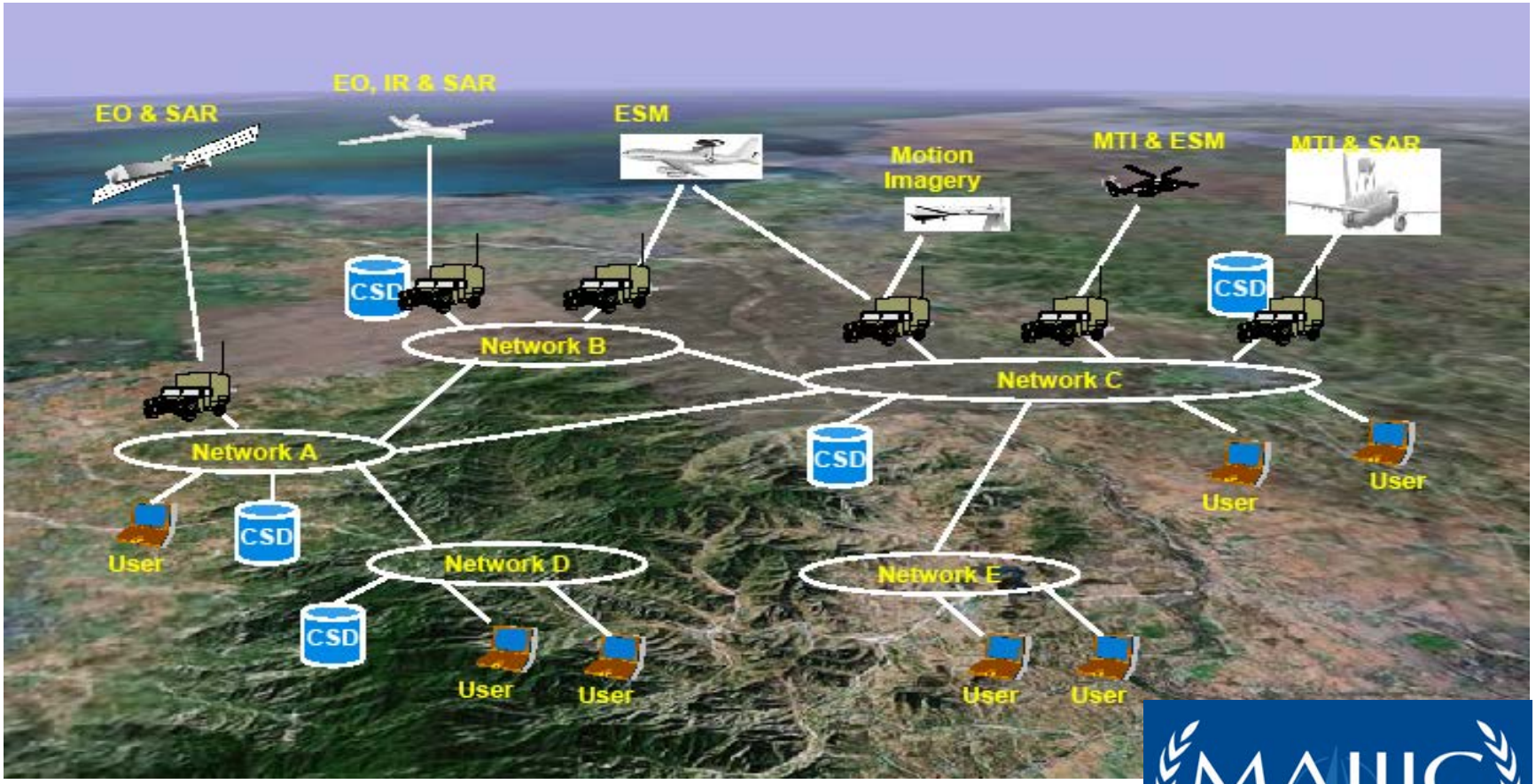
backbone of a ground picture: moving target tracks

- airborne, dislocated, mobile sensor platforms
- vehicles, ships, ‘low-flyers’, radars, convoys
- occlusions: Doppler-blindness, topography
- road maps, terrain information, tactical rules
- dense target / dense clutter situations: MHT

Examples of GMTI Tracks (live exercise)



Ground Situational Awareness for ISR Coalitions



NEC: Network Enabled Capabilities

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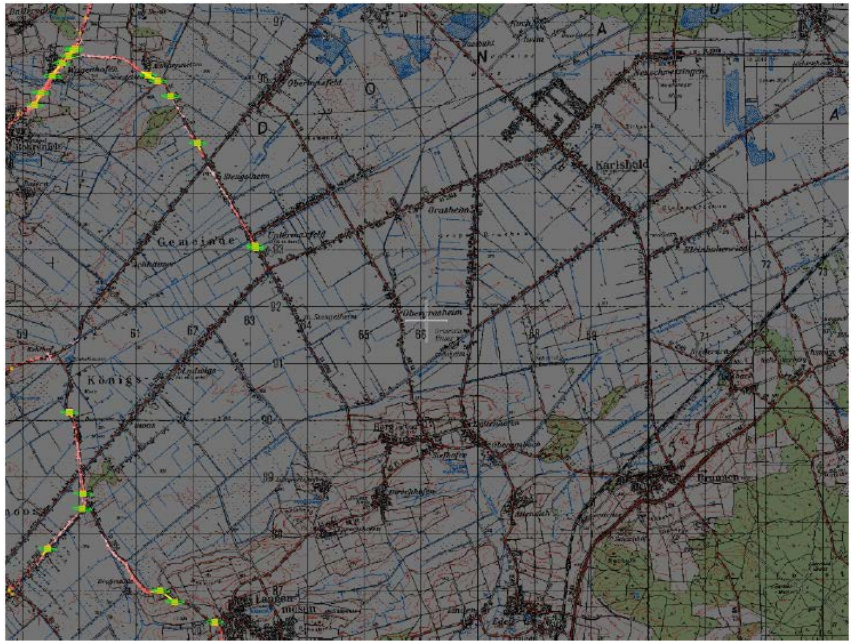
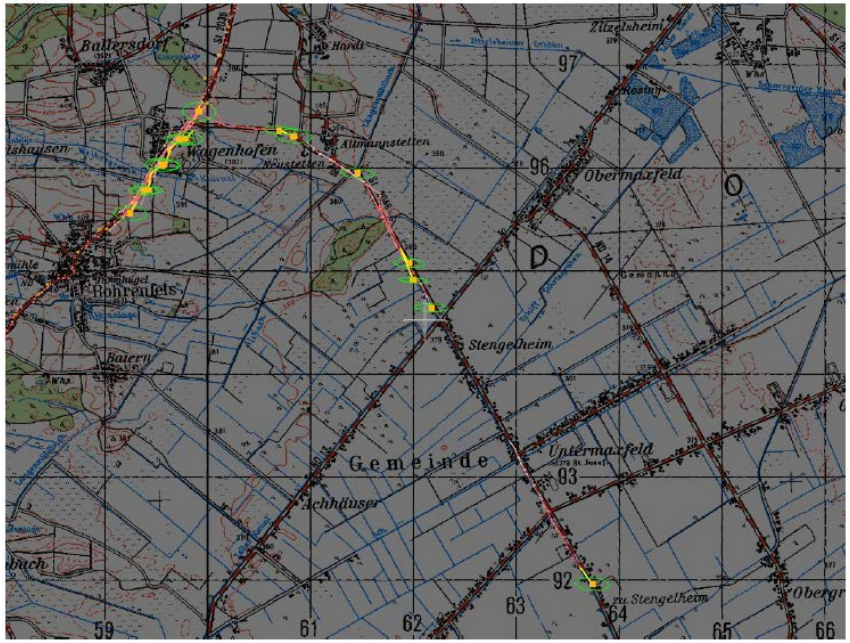
Multiple Target Tracking:MAJIIC-Expamles





Automated track-before-detect, maintenance, target number estimation

Map data: fast extraction, precision, continuity, off- / on-road classification,…

GMTI: Fusion of ‘Negative’ Information

low-DOPPLER targets can be masked by the GMTI clutter notch

- *fading:* series of missing plots (target/sensor geometry)
- *stopping targets:* indistinguishable from ground clutter
- *mdv:* minimum detectable velocity (sensor parameter)

a simple GMTI detection model: qualitative discussion

- detection depends on target kinematics & target/sensor geometry
- detection probability $P_D(\mathbf{x}_k)$ is small if $n_c = \dot{r}_k - \dot{r}_{\mathrm{C}}(\mathbf{x}_k) < \mathit{mdv}$
- there exists a narrow transition region between these domains

sensor performance: *quantitative* model

- **basis:** $\mathrm{snir} = \mathrm{snir}(r_k, \varphi_k, \dot{r}_k)$**, Signal-to-Noise+Interference Ratio**

$$\mathrm{snir} = \mathrm{snir}_0 \underbrace{\left(\frac{\bar{\sigma}_k}{\sigma_0}\right)}_{\text{rcs}} \underbrace{\left(\frac{r_k}{r_0}\right)^{-4}}_{\text{propagation}} \underbrace{D(\varphi_k)}_{\text{directivity}} \underbrace{\left[1 - \mathrm{e}^{-\log 2 \left(\frac{n_c(r_k, \varphi_k, \dot{r}_k)}{v_m}\right)^2}\right]}_{\text{clutter notch } (<\frac{1}{2} \text{ für } |n_c| < v_m)}$$

- **quadrature detector with given** P_{FA}**, rcs fluctuations: SWERLING I**

$$P_d(r_k, \varphi_k, \dot{r}_k) = P_{\mathrm{FA}}^{\frac{1}{1+\mathrm{snir}(r_k, \varphi_k, \dot{r}_k)}}$$

- **as usual: residual clutter; bias free, GAUSSian errors (monopulse)**

$$\sigma_{r,\varphi,\dot{r}}(r_k, \varphi_k, \dot{r}_k) = \Sigma_{r,\varphi,\dot{r}} / \sqrt{\mathrm{snir}(r_k, \varphi_k, \dot{r}_k)}$$

filtering: $p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \xrightarrow[\text{sensor modell}]{\text{sensor data } Z_k} p(\mathbf{x}_k|\mathcal{Z}^k)$

$$p(\mathbf{x}_k|\mathcal{Z}^k) = \frac{p(Z_k, n_k|\mathbf{x}_k)\, p(\mathbf{x}_k|\mathcal{Z}^{k-1})}{\int d\mathbf{x}_k\, \underbrace{p(Z_k, n_k|\mathbf{x}_k)}_{\text{likelihood}}\, \underbrace{p(\mathbf{x}_k|\mathcal{Z}^{k-1})}_{\text{prediction}}} \qquad \textbf{(BAYES)}$$

likelihood (depending on sensor data, modeling parameters):

$$p(Z_k, n_k|\mathbf{x}_k) \propto \pi_0^0 + \sum_{n=0}^{n_k} \sum_{i=0}^{1} \pi_n^i\, \mathcal{N}\Big(\mathbf{z}_k^{ni};\, \mathbf{H}_{k|k-1}^{ni}\mathbf{x}_k,\, \mathbf{R}_{k|k-1}^{ni}\Big)_{n\neq 0, i\neq 0}$$

(after some calculations and mild approximations; π_n^i constant coefficients)

essential: exploit state dependency of $P_d(\mathbf{x}_k)$!

Clutter Notch: A Priori Information!

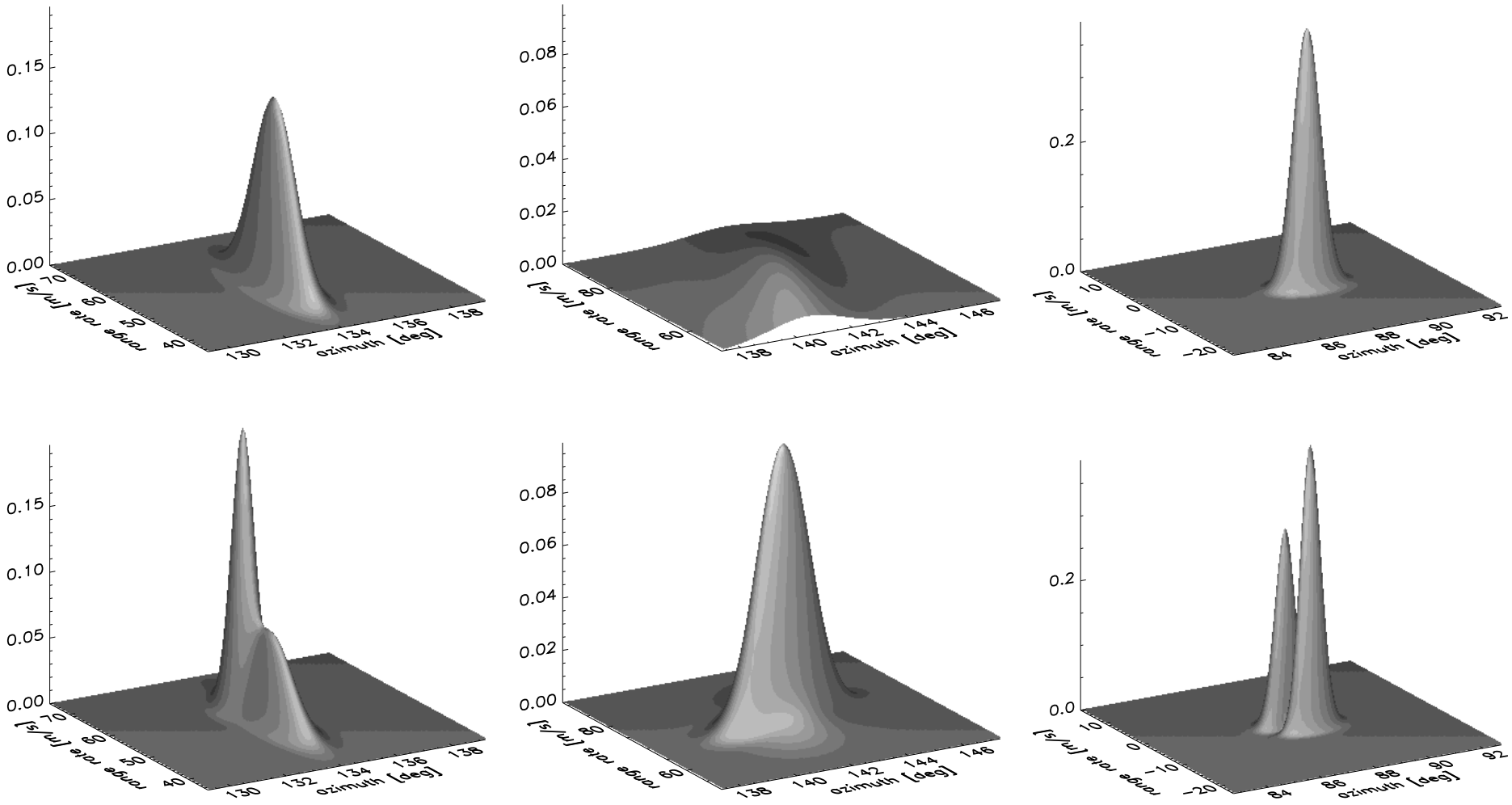
- *current* position (Sensor-to-target-geometry)
- sensor specific *width* (STAP $\rightarrow$ MDV)
- detection process: *generic* model

GMTI model $\Rightarrow$ mixture densities:

- well-understood formalism directly applicable
- class of GAUSS*ian mixtures* remains invariant
- pdfs characterized by a *set of parameters*
- growing memory: standard-type approximations

model inherent: **reason for missing detections**
$\Rightarrow$ An adequate treatment becomes possible!

PDFs with / without exploiting the GMTI sensor model

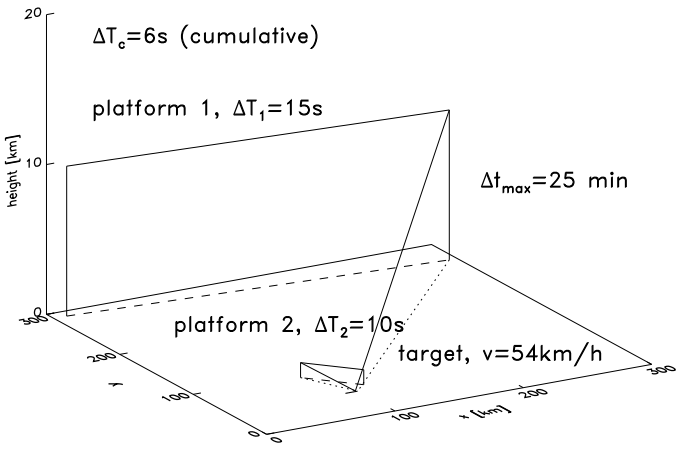


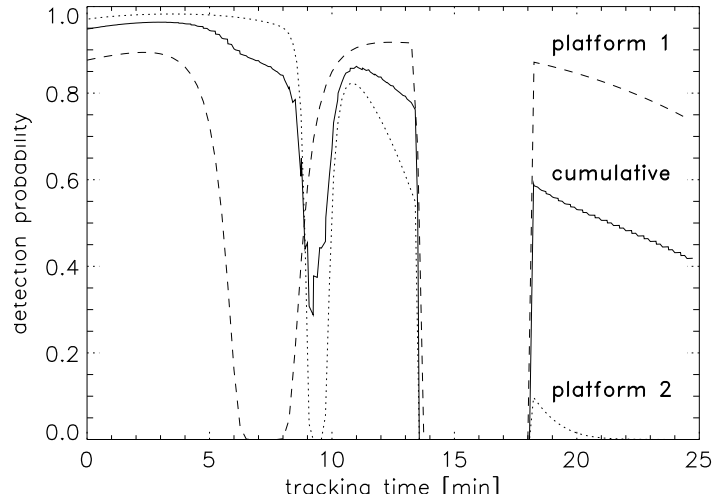
Missing detection occurred near the clutter notch

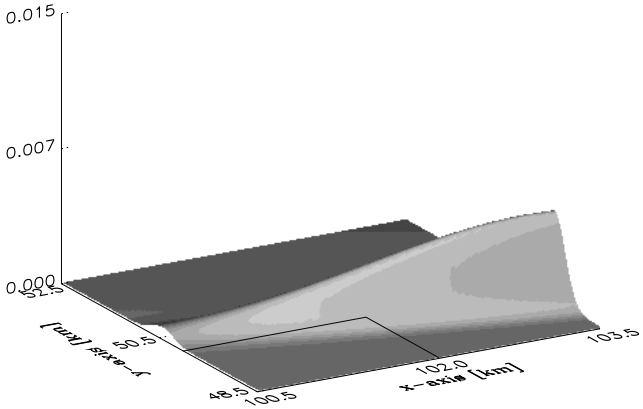
Several missing detections in the clutter notch

Detection occurred near the clutter notch

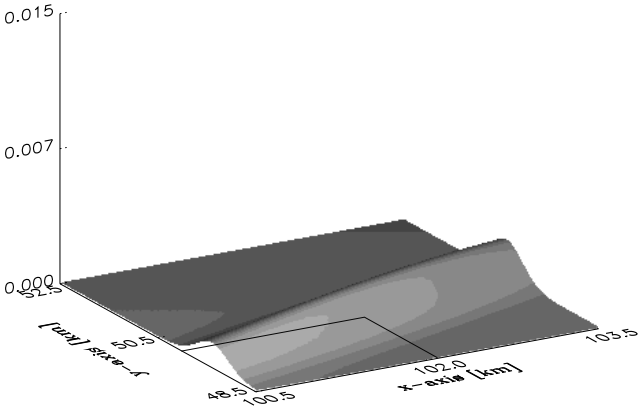
early detection of stopping targets



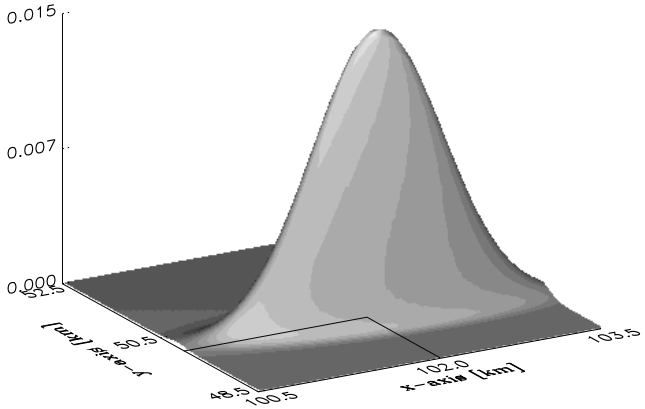




neg. output sensor 1



neg. output sensor 2



fusion: sensor 1+2

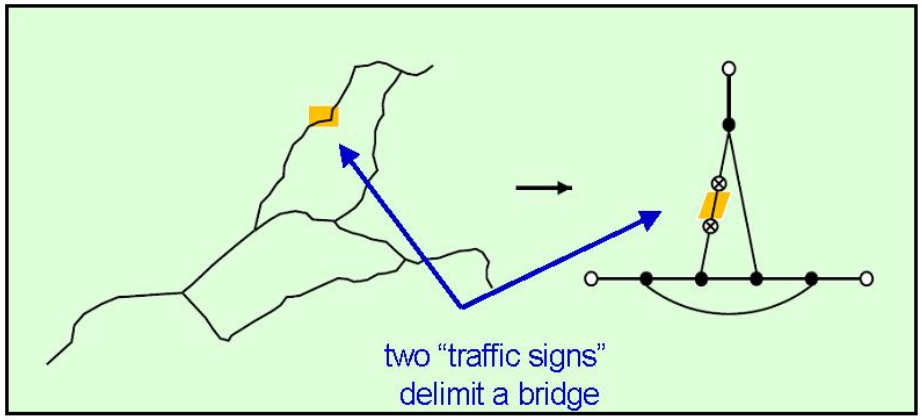
A *'negative' sensor output* can also provide information on the kinematical state vector of a target.

- ***fictitious plot*: function of position / radial speed**
- ***mdv*: appears as a fictitious measurement error**
- ***fusion*: exploit differing target/sensor geometries**

Koch, W., Ground Target Tracking with STAP Radar: Selected Tracking Aspects. *Chapter 14 in: Klemm, R. (Ed.): Applications of Space-time Adaptive Processing. Institution of Electrical Engineers, IEE Press, London (2004)*.

Koch, W. and Klemm, R., Ground Target Tracking with STAP Radar. *IEE Proceedings on Radar, Sonar and Navigation, Vol. 148, No. 3, p.173-185, June 2001*.

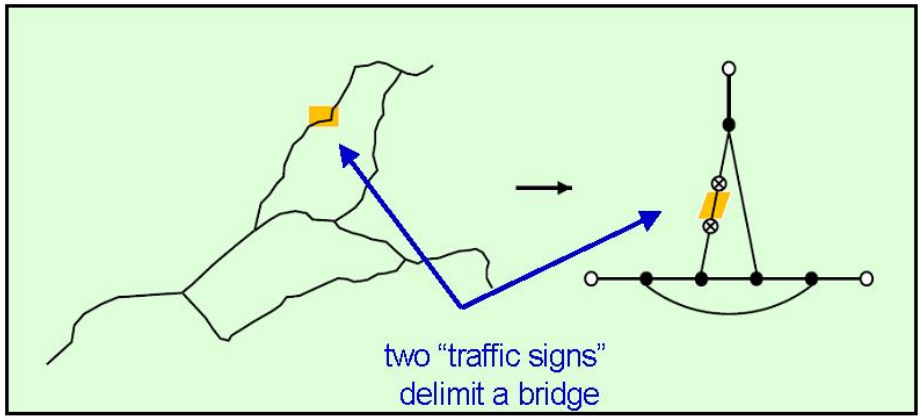
Road-map Assisted Vehicle Tracking



road network represented by a graph:

nodes: sources, crossings, ‘traffic signs’
edges: representation of road segments

Road-map Assisted Vehicle Tracking



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nodes: sources, crossings, 'traffic signs'
edges: representation of road segments

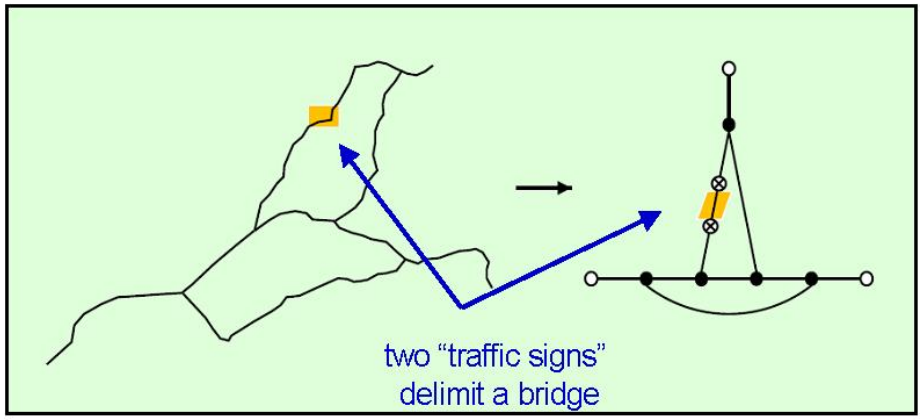
road maps: polygons $\mathcal{R}(l)$ (arc length l, with mapping/discretization errors)

road coordinates: vehicle kinematics described by: $\boxed{\mathbf{x}_k^r = (l_k, \dot{l}_k)^\top}$ (2D!)

dynamics model: prediction in road coordinates: $p(\mathbf{x}_{k-1}^r|\mathcal{Z}^{k-1}) \xrightarrow{\text{Dyn.}} p(\mathbf{x}_k^r|\mathcal{Z}^{k-1})$

sensor model: filtering step in sensor coordinates: $p(\mathbf{x}_k^s|\mathcal{Z}^{k-1}) \xrightarrow[Z_k]{\text{Sen.}} p(\mathbf{x}_k^s|\mathcal{Z}^k)$

Road-map Assisted Vehicle Tracking



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$$\underbrace{p(\mathbf{x}_k^r|\mathcal{Z}^{k-1})}_{\text{road coordinates}} \xrightarrow[\text{mapping errors } \mathbf{R}_i^j]{\text{road network}} \underbrace{p(\mathbf{x}_k^s|\mathcal{Z}^{k-1})}_{\text{sensor coordinates}}$$

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$$\underbrace{p(\mathbf{x}_k^r|\mathcal{Z}^{k-1})}_{\text{road coordinates}} \xrightarrow[\text{mapping errors } \mathbf{R}_i^j]{\text{road network}} \underbrace{p(\mathbf{x}_k^s|\mathcal{Z}^{k-1}) = \sum_{m=1}^{n_r} p(\mathbf{x}_k^s|m, \mathcal{Z}^{k-1})\, p(m|\mathcal{Z}^{k-1})}_{\text{sensor coordinates}}$$

calculations/mild approximations yield:

- **result: GAUSS*ian mixtures* referring to road segments**
- **road segments: artificial *scalar measurements* + errors**
- **seamless embedding into BAYES*ian & PMHT formalisms***
- **road *networks* imply inherent *multihypothesis* structure**

vice versa: tracking-based road map extraction

high-precision, up-to-date roads; possibly: sensor registration

Ulmke, M., Koch, W., Road-map Extraction using GMTI Tracking. *9th International Conference on Information Fusion, Florence, July 2006*.

Koch, W., Koller, J., Ulmke, M., Ground Target Tracking and Road Map Extraction. *ISPRS Journal of Photogrammetry & Remote Sensing, 61 (2006), 197-208, Elsevier*.

Ulmke, M., Koch, W., Road-Map Assisted Ground Moving Target Tracking *IEEE Transactions on Aerospace and Electronic Systems, Vol. 42, No. 4, p.1264-1274, October 2006*.

1. **Multiple Hypothesis Tracking: Basic Principles**

2. **Tracking & Management for Phased Array Radar**

3. **GMTI Tracking and Fusion of Context Information**